

# Tensor polarization dependent fragmentation functions and $e^+e^- \rightarrow V\pi X$ at high energies

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We present the systematic results for three dimensional fragmentation functions of spin one hadrons defined via quark-quark correlator. There are totally 72 such fragmentation functions, among them 18 are twist-2, 36 are twist-3 and 18 are twist-4. We also present the relationships between the twist-3 parts and those defined via quark-gluon-quark correlator obtained from the QCD equation of motion. We show that two particle semi-inclusive hadron production process  $e^+e^- \rightarrow V\pi X$  at high energies is one of the best places to study the three-dimensional tensor polarization dependent fragmentation functions. We present the general kinematic analysis of this process and show that the cross section should be expressed in terms of 81 independent structure functions. After that we present parton model results for the hadronic tensor, the structure functions, the azimuthal and spin asymmetries in terms of these gauge invariant fragmentation functions at the leading order pQCD up to twist-3.

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## I. INTRODUCTION

In describing high energy reactions, we need two sets of important quantities, the parton distribution functions (PDFs) and the fragmentation functions (FFs). The former is used to describe the hadron structure and the latter describes the hadronization process. In a quantum field theoretical formulation, both PDFs and FFs are defined via the corresponding quark-quark correlator. The quark-quark correlator is defined as a matrix in the Dirac space depending on the hadron states. It is then decomposed into different components expressed in terms of the basic Lorentz covariants and the scalar functions. These scalar functions contain the information of the hadron structure and/or hadronization mechanism and are called the corresponding PDFs or FFs. In many cases in literature, specific PDFs and/or FFs are introduced whenever needed, sometimes with different conventions and/or notations. With the development of the related studies, it is necessary and useful to make a systematic study and present a complete set of such results. The results for three-dimensional PDFs of the nucleon defined in this way are presented in [1] in a systematical way. Since usually different types of hadrons with different flavors and spins are produced in a high energy reaction, FFs are therefore more involved and perhaps even more interesting but less studied yet. Specific recent discussions can also be found e.g. in [2–17]. A short summary can be found in a recent unpublished note and short reviews [18–20].

In this paper, we summarize the results for three dimensional FFs defined via quark-quark correlator for spin one hadrons in a systematical way. The FFs are divided into a spin independent part, a vector polarization dependent part and a tensor polarization dependent part. Formerly, the spin independent part is the same as those for spin zero hadrons and the vector polarization dependent part is the same as those for spin-1/2 hadrons. They are also similar to those for PDFs presented e.g. in [1] for the corresponding cases. We will pay special attention to the tensor polarization dependent part including higher twist contributions. In this connection, we will

in particular show also FFs defined via the quark-gluon-quark correlator and their relationships to those defined via quark-quark correlator obtained using Quantum Chromodynamics (QCD) equation of motion.

The most convenient place to study the three dimensional FFs of vector mesons is perhaps  $e^+e^- \rightarrow V\pi X$ . We present the results for the general kinematical analysis of this process and calculate the hadronic tensor and differential cross section up to twist-3 at leading order in perturbative QCD. We also present the results for the tensor polarizations of  $V$  in terms of the three dimensional FFs.

The rest of the paper is organized as follows. After this introduction, we briefly summarize the general procedure of deriving the results of FFs from the quark-quark correlator and present results and relationships to those defined via quark-gluon-quark correlator at twist-3 in Sec. II. We make general kinematical analysis of  $e^+e^- \rightarrow V\pi X$  in Sec. III. We calculate the hadronic tensor at leading order perturbative QCD up to twist-3 in Sec. IV. We present the results for the structure functions in Sec. V and those for azimuthal and spin asymmetries in Sec. VI. We make a summary and a discussion in Sec. VII. Since most of the equations are rather long, we will present the discussions in the corresponding sections but show most of the formulae and tables in the appendices.

## II. FRAGMENTATION FUNCTIONS DEFINED VIA QUARK-QUARK CORRELATOR

A systematic analysis is given in a recent unpublished note [18]. For completeness, we briefly summarize the basic ideas in this section and summarize the results in appendix A. Similar to parton distribution and/or correlation functions, in quantum field theory, the quark fragmentation is defined via the quark-quark correlator given by,

$$\hat{\Xi}_{ij}^{(0)}(k_F; p, S) = \frac{1}{2\pi} \sum_X \int d^4\xi e^{-ik_F\xi} \langle 0 | \mathcal{L}^\dagger(0; \infty) \psi_i(0) | p, S; X \rangle$$

$$\times \langle p, S; X | \bar{\psi}_j(\xi) \mathcal{L}(\xi; \infty) | 0 \rangle, \quad (2.1)$$

where  $k_F$  and  $p$  denote the 4-momenta of the quark and the hadron respectively,  $S$  denotes the spin of the hadron;  $\mathcal{L}(\xi; \infty)$  is the gauge link that is given by,

$$\begin{aligned} \mathcal{L}(\xi, \infty) &= P e^{ig \int_{\xi^-}^{\infty} d\eta^- A^+(\eta^-; \xi^+, \vec{\xi}_\perp)} \\ &= 1 + ig \int_{\xi^-}^{\infty} d\eta^- A^+(\eta^-; \xi^+, \vec{\xi}_\perp) \\ &+ (ig)^2 \int_{\xi^-}^{\infty} d\eta_1^- \int_{\xi^-}^{\eta_1^-} d\eta_2^- A^+(\eta_2^-; \xi^+, \vec{\xi}_\perp) A^+(\eta_1^-; \xi^+, \vec{\xi}_\perp) \\ &+ \dots \end{aligned} \quad (2.2)$$

The correlator given by Eq. (2.1) satisfies the following constraints imposed by hermiticity and parity conservation, i.e.,

$$\hat{\Xi}^{\dagger(0)}(k_F; p, S) = \gamma^0 \hat{\Xi}^{(0)}(k_F; p, S) \gamma^0, \quad (2.3)$$

$$\hat{\Xi}^{(0)}(k_F; p, S) = \gamma^0 \hat{\Xi}^{(0)}(k_F^{\mathcal{P}}; p^{\mathcal{P}}, S^{\mathcal{P}}) \gamma^0, \quad (2.4)$$

where a vector with the superscript  $\mathcal{P}$  denotes the result after space reflection such as  $p_\mu^{\mathcal{P}} = p^\mu$ . Unlike that for hadron structure, because of the presence of the gauge link and final state interactions between  $h$  and  $X$ , time reversal invariance puts no such simple constraint on the correlator  $\hat{\Xi}^{(0)}(k_F; p, S)$ .

The three-dimensional or the transverse momentum dependent (TMD) FFs are defined via the three-dimensional quark-quark correlator  $\hat{\Xi}^{(0)}(z, k_{F\perp}; p, S)$  obtained from  $\hat{\Xi}^{(0)}(k_F, p, S)$  by integrating over  $k_F^-$ , i.e.,

$$\begin{aligned} \hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) &= \sum_X \int \frac{p^+ d\xi^-}{2\pi} d^2\xi_\perp e^{-i(p^+ \xi^- / z - \vec{k}_{F\perp} \cdot \vec{\xi}_\perp)} \\ &\times \langle 0 | \mathcal{L}^\dagger(0; \infty) \psi(0) | p, S; X \rangle \langle p, S; X | \bar{\psi}(\xi) \mathcal{L}(\xi; \infty) | 0 \rangle, \end{aligned} \quad (2.5)$$

where  $z = p^+ / k_F^+$  is the longitudinal momentum fraction defined in light cone coordinates. Here we use the light-cone coordinate and define the light-cone unit vectors as  $\bar{n} = (1, 0, \vec{0}_\perp)$ ,  $n = (0, 1, \vec{0}_\perp)$  and  $n_\perp = (0, 0, \vec{1}_\perp)$ . We choose the hadron's momentum as  $z$ -direction so that  $p$  is decomposed as  $p^\mu = p^+ \bar{n}^\mu + (M^2/2p^+) n^\mu$ .

The FFs are obtained from  $\hat{\Xi}^{(0)}(z, k_{F\perp}; p, S)$  by decomposing it in the following two steps. First, we note that  $\hat{\Xi}^{(0)}(z, k_{F\perp}; p, S)$  is a matrix in Dirac space and expand it in terms of the  $\Gamma$ -matrices,  $\Gamma = \{\mathbf{1}, i\gamma_5, \gamma^\alpha, \gamma_5 \gamma^\alpha, i\sigma^{\alpha\beta} \gamma_5\}$ , i.e.,

$$\begin{aligned} \hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) &= \Xi^{(0)}(z, k_{F\perp}; p, S) + i\gamma_5 \tilde{\Xi}^{(0)}(z, k_{F\perp}; p, S) \\ &+ \gamma^\alpha \Xi_\alpha^{(0)}(z, k_{F\perp}; p, S) + \gamma_5 \gamma^\alpha \tilde{\Xi}_\alpha^{(0)}(z, k_{F\perp}; p, S) \\ &+ i\sigma^{\alpha\beta} \gamma_5 \Xi_{\alpha\beta}^{(0)}(z, k_{F\perp}; p, S). \end{aligned} \quad (2.6)$$

The coefficient functions are given by,

$$\begin{aligned} \Xi^{(0)[\Gamma]}(z, k_{F\perp}; p, S) &= \frac{1}{4} \sum_X \int \frac{p^+ d\xi^-}{2\pi} d^2\xi_\perp e^{-i(p^+ \xi^- / z - \vec{k}_{F\perp} \cdot \vec{\xi}_\perp)} \\ &\times \langle p, S; X | \bar{\psi}(\xi) \mathcal{L}(\xi; \infty) | 0 \rangle \Gamma^{(0)} \langle 0 | \mathcal{L}^\dagger(0; \infty) \psi(0) | p, S; X \rangle, \end{aligned} \quad (2.7)$$

where  $\Xi^{(0)[\Gamma]}$  represents respectively  $\Xi^{(0)}$ ,  $\tilde{\Xi}^{(0)}$ ,  $\Xi_\alpha^{(0)}$ ,  $\tilde{\Xi}_\alpha^{(0)}$  and  $\Xi_{\alpha\beta}^{(0)}$  for different  $\Gamma$ 's. Together with the demands imposed by

the hermiticity and parity invariance [Eqs. (2.3) and (2.4)], the Lorentz invariance demands that all the corresponding coefficient functions are real and are Lorentz scalar, pseudo-scalar, vector, axial-vector and tensor respectively. Furthermore, the tensor  $\Xi_{\alpha\beta}^{(0)}$  is anti-symmetric in Lorentz indices and odd under space reflection which implies that it can be made out of a vector and an axial vector.

Second, we expand these coefficient functions according to their respective Lorentz transformation properties in terms of the basic Lorentz covariants constructed from basic variables at hand. They are expressed as the sum of the basic Lorentz covariants multiplied by scalar functions of  $z$  and  $k_{F\perp}^2$ . These scalar functions are the three-dimensional FFs. We note in particular that because of the hermiticity given by Eq. (2.3), these FFs defined via quark-quark correlator are real.

Clearly, the basic Lorentz covariants that we can construct depend on what basic variable(s) that we have at hand. Besides the four-momenta  $p$  and  $k_F$ , we have the variables describing the spin states. Such variables are different for hadrons with different spins. For spin-1 hadrons, the polarization is described by a  $3 \times 3$  density matrix  $\rho$ , which, in the rest frame of the hadron, is usually decomposed as [21],

$$\rho = \frac{1}{3} (\mathbf{1} + \frac{3}{2} S^i \Sigma^i + 3 T^{ij} \Sigma^{ij}), \quad (2.8)$$

where,  $\Sigma^i$  is the spin operator of spin-1 particle, and  $\Sigma^{ij} = \frac{1}{2} (\Sigma^i \Sigma^j + \Sigma^j \Sigma^i) - \frac{2}{3} \delta^{ij}$ . The spin polarization tensor  $T^{ij} = \text{Tr}(\rho \Sigma^{ij})$ , and is parameterized as,

$$\mathbf{T} = \frac{1}{2} \begin{pmatrix} -\frac{2}{3} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3} S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3} S_{LL} \end{pmatrix}. \quad (2.9)$$

Here, besides the polarization vector  $S$ , we also need a polarization tensor  $T$ . The polarization vector  $S$  is similar to that for spin-1/2 hadrons and the tensor  $T$  has five independent components that are given by a Lorentz scalar  $S_{LL}$ , a Lorentz vector  $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$  and a Lorentz tensor  $S_{TT}^{\mu\nu}$  that has two nonzero independent components  $S_{TT}^{xx} = -S_{TT}^{yy}$  and  $S_{TT}^{xy} = S_{TT}^{yx}$ . In a covariant form, the polarization vector  $S$  is decomposed as,

$$S^\mu = \lambda \frac{p^+}{M} \bar{n}^\mu + S_T^\mu - \lambda \frac{M}{2p^+} n^\mu, \quad (2.10)$$

where  $\lambda$  denotes the helicity and  $S_T = (0, 0, \vec{S}_T)$  denotes the transverse polarization. The tensor polarization  $T^{\mu\nu}$  is expressed as [21],

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{2} \left[ \frac{4}{3} S_{LL} \left( \frac{p^+}{M} \right)^2 \bar{n}^\mu \bar{n}^\nu + \frac{p^+}{M} n^{[\mu} S_{LT}^{\nu]} - \frac{2}{3} S_{LL} (\bar{n}^{[\mu} n^{\nu]} - g_\perp^{\mu\nu}) \right. \\ &\left. + S_{TT}^{\mu\nu} - \frac{M}{2p^+} \bar{n}^{[\mu} S_{LT}^{\nu]} + \frac{1}{3} S_{LL} \left( \frac{M}{p^+} \right)^2 n^\mu n^\nu \right], \end{aligned} \quad (2.11)$$

where we used the anti-commutation symbol  $A^{[\mu} B^{\nu]} \equiv A^\mu B^\nu - A^\nu B^\mu$ , and also in the following of this paper  $A^{[\mu} B^{\nu]} \equiv A^\mu B^\nu - A^\nu B^\mu$ , and  $g_\perp^{\mu\nu} \equiv g^{\mu\nu} - \bar{n}^\mu n^\nu - n^\mu \bar{n}^\nu$ .

Hence, for spin-1 hadrons, the quark-quark correlator  $\hat{\Xi}^{(0)}$  can be written as the sum of a polarization independent part

$\hat{\Xi}^{U(0)}$ , a vector polarization dependent part  $\hat{\Xi}^{V(0)}$  and a tensor polarization dependent part  $\hat{\Xi}^{T(0)}$ , i.e.,

$$\hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) = \hat{\Xi}^{U(0)}(z, k_{F\perp}; p) + \hat{\Xi}^{V(0)}(z, k_{F\perp}; p, S) + \hat{\Xi}^{T(0)}(z, k_{F\perp}; p, S). \quad (2.12)$$

Since the polarization dependence is linear to the corresponding spin parameters, formally, the spin independent part is exactly the same as that for spin-0 hadrons, the vector polarization dependent part is the same as that for spin-1/2 hadrons. The tensor polarization dependent part is new and contributes only for spin-1 hadron production. We summarize them separately in the following.

Before we present the results, we describe the notation system for the FFs used through out the paper. We will use  $D$ ,  $G$  and  $H$  for unpolarized, longitudinally polarized and transversely polarized quarks. They correspond to those FFs obtained via decompositions of the vector, axial-vector and tensor part of the correlator. Those defined via the scalar and the pseudo-scalar are denoted by  $E$ . A number  $j$  in the subscripts specifies the twist:  $j = 1$  for twist-2, null (no number) for twist-3 and  $j = 3$  for twist-4. We will also use different symbols in the subscripts to denote the polarization of the produced hadron such as  $L$  and  $T$  in the vector polarization case and  $LL$ ,  $LT$  or  $TT$  in the tensor polarization case; a  $\perp$  in the superscript denotes that the corresponding basic Lorentz covariant is  $k_{F\perp}$ -dependent.

If we decompose the quark field in Eq. (2.7) into the sum of the right- and left-handed parts, i.e.,  $\psi = \psi_R + \psi_L$  with  $\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi$ . We see that for  $\Gamma = \mathbf{I}$ ,  $i\gamma_5$  and  $i\sigma^{\alpha\beta}\gamma_5$ ,  $\bar{\psi}_R\Gamma\psi_L$  and  $\bar{\psi}_L\Gamma\psi_R$  are non-zero. So the terms related to them (i.e., the  $E$  and  $H$ -terms) correspond to helicity-flipped quark structure and are called chiral-odd ( $\chi$ -odd). Similarly, for  $\Gamma = \gamma^\alpha$  and  $\gamma^5\gamma^\alpha$ ,  $\bar{\psi}_L\Gamma\psi_L$  and  $\bar{\psi}_R\Gamma\psi_R$  are non-zero. Hence, the terms related to them (i.e. the  $D$ 's and the  $G$ 's) do not flip the quark helicity and are  $\chi$ -even. We also recall the properties of the fermion bilinears under time-reversal  $\hat{T}$ , i.e.,

$$\begin{aligned} & \hat{T} \{ \bar{\psi}\psi, \bar{\psi}i\gamma_5\psi, \bar{\psi}\gamma_\alpha\psi, \bar{\psi}\gamma_5\gamma_\alpha\psi, \bar{\psi}i\sigma_{\alpha\beta}\gamma_5\psi \} \\ & \Rightarrow \{ \bar{\psi}\psi, -\bar{\psi}i\gamma_5\psi, \bar{\psi}\gamma^\alpha\psi, \bar{\psi}\gamma_5\gamma^\alpha\psi, \bar{\psi}i\sigma^{\alpha\beta}\gamma_5\psi \}. \end{aligned} \quad (2.13)$$

Using this, we can determine whether a component of FF defined via quark-quark correlator is time reversal even (T-even) or odd (T-odd) according to the time reversal behavior of the corresponding basic Lorentz covariant. However, we should also note that they are usually referred as ‘‘naive T-odd’’ or ‘‘naive T-even’’ because the interactions between the produced hadron  $h$  and the rest  $X$  can destroy simple regularities so all of them can exist in a practical hadronization process.

## A. Results of the decomposition and FFs

### 1. The unpolarized part

For the spin independent part  $\hat{\Xi}^{U(0)}(z, k_{F\perp}; p)$ , the independent variables that can be used to construct the basic Lorentz

covariants are  $p_\alpha$ ,  $k_{F\perp\alpha}$ , and  $n_\alpha$ . The basic Lorentz covariants that we can construct from them are: one Lorentz scalar  $p^2 = M^2$ , no pseudo-scalar, three Lorentz vectors,  $p$ ,  $k_{F\perp}$  and  $n$ , one axial vector  $\varepsilon_{\perp\rho\alpha}k_{F\perp}^\rho \equiv \tilde{k}_{F\alpha}$ , and three anti-symmetric and space reflection odd Lorentz tensors  $p_{[\rho}\tilde{k}_{F\perp\alpha]}$ ,  $\varepsilon_{\perp\rho\alpha}$  and  $n_{[\rho}\tilde{k}_{F\perp\alpha]}$ . Here  $\varepsilon_{\perp\rho\alpha} = \varepsilon_{\mu\nu\rho\alpha}\tilde{n}^\mu n^\nu$  and  $\varepsilon_{\mu\nu\rho\alpha}$  is the anti-symmetric tensor. We also use the notation  $\tilde{a}_{\perp\mu} \equiv \varepsilon_{\perp\rho\mu}a^\rho$  to denote the transverse vector perpendicular to  $a_\perp$ , and note in particular that  $\tilde{a}_\perp \cdot b_\perp = \varepsilon_{\perp\rho\sigma}a^\rho b^\sigma = -a_\perp \cdot \tilde{b}_\perp$ , and  $\tilde{\tilde{a}}_\perp = -a_\perp$ . The general decomposition of the spin independent part of the quark-quark correlator is given by Eqs. (A1-A5) in Appendix A. We obtain 8 unpolarized TMD FFs, 2 of them contribute at twist-2, 4 at twist-3 and the other 2 at twist-4 level.

From Eqs. (A1-A5), we see in particular the existence of a leading twist FF  $H_1^\perp(z, k_{F\perp})$  that leads to azimuthal asymmetry of produced hadron in fragmentation of a transversely polarized quark. This was first introduced in [4] and is now known as Collins function. We see also a twist-4 addendum to it described by  $H_3^\perp(z, k_{F\perp})$ .

If we integrate over  $d^2k_{F\perp}$ , we obtain the one dimensional results as given by Eqs. (A6-A8) in the appendix. We see that there are only 4 left and the number density  $D_1(z)$  is the only leading twist, 2 of them contribute at twist-3 and the other one at twist-4.

We note in particular the direct one to one correspondence between the results obtained in this case for FFs and those obtained in [1] for PDFs. The only obvious difference is the existence of the naive time reversal odd  $H(z)$  due to final interaction between  $h$  and  $X$  while the corresponding term vanishes for PDFs.

### 2. The vector polarization dependent part

For the vector polarization dependent part, we have, besides  $p_\alpha$ ,  $k_{F\perp\alpha}$ , and  $n_\alpha$ , the polarization vector  $S$  to use to construct the basic Lorentz covariants. The results obtained are given by Eqs. (A10-A14) in Appendix A. We see that there are 24 vector polarization dependent TMD FFs, 6 of them contribute at twist-2, 12 at twist-3 and the other 6 at twist-4 level. Among them, 8 are naive T-odd ( $E_T^\perp$ ,  $E_L$ ,  $E_T'^\perp$ ,  $D_{1T}^\perp$ ,  $D_L^\perp$ ,  $D_T$ ,  $D_T^\perp$  and  $D_{3T}^\perp$ ), and the other 16 are T-even.

We also note that 4 of them ( $E_L$ ,  $G_{1L}$ ,  $G_L^\perp$ ,  $G_{3L}$ ) are for longitudinal (to longitudinal) spin transfer; 6 of them ( $H_{1T}$ ,  $H_{1T}^\perp$ ,  $H_T^\perp$ ,  $H_{3T}$ ,  $H_{3T}^\perp$ ) are for transverse (to transverse) spin transfer; 5 of them ( $E_T'^\perp$ ,  $G_{1T}^\perp$ ,  $G_T$ ,  $G_T^\perp$ ,  $G_{3T}$ ) are for longitudinal to transverse spin transfer; 3 of them ( $H_{1L}^\perp$ ,  $H_L$ ,  $H_{3L}^\perp$ ) are for transverse to longitudinal spin transfer; and the other 6 ( $E_T^\perp$ ,  $D_{1T}^\perp$ ,  $D_T$ ,  $D_L^\perp$ ,  $D_T^\perp$ ,  $D_{3T}^\perp$ ) are for induced polarizations which leads to hadron polarizations in fragmentation of unpolarized quark. At leading twist, we have a  $D_{1T}^\perp$  for induced polarization, a longitudinal ( $G_{1L}$ ), two transverse ( $H_{1T}$ ,  $H_{1T}^\perp$ ), a longitudinal to transverse ( $G_{1T}^\perp$ ) and a transverse to longitudinal ( $H_{1L}^\perp$ ) spin transfer.

We note in particular the induced polarization terms described by  $E_T^\perp$  and the  $D$ 's in fragmentation of an unpolarized quark. At leading twist, there is a Sivers type [22] FF  $D_{1T}^\perp$  describing polarization transverse to the production plane

and corresponding to the transverse hyperon polarizations observed in high energy hadron-hadron and hadron-nucleus collisions [23]. Other higher twist FFs describe polarizations in longitudinal as well as two transverse directions.

If we integrate over  $d^2k_{F\perp}$ , we obtain the results given by Eqs. (A15-A19) in Appendix A. We see that only 8 terms survive, which means that, in the one-dimensional case, for the vector polarization dependent part, we have totally 8 FFs. We see also that 2 of them are leading twist, they are the longitudinal spin transfer  $G_{1L}(z)$  and the transverse spin transfer  $H_{1T}(z)$ . We also have 4 twist-3 FFs that lead to induced polarization of hadron and 2 twist-4 FFs that are addenda to the longitudinal and transverse spin transfer respectively. We also see that in this case induced polarization in the transverse direction exists at twist-3.

We note again the direct one to one correspondence between the results obtained in this case and those obtained in [1] for PDFs. The difference is the existence of the naive time reversal odd  $E_L(z)$  and  $D_T(z)$  due to final state interactions between  $h$  and  $X$  while the corresponding term vanishes for PDFs. While  $E_L(z)$  is an addendum to  $G_{1L}(z)$ ,  $D_T(z)$  leads to transverse polarization in fragmentation of unpolarized quark. Both of them contribute at twist-3.

### 3. The tensor polarization dependent part

The general decomposition of the tensor polarization dependent part is given by Eqs. (A21-A25) in Appendix A which is obtained by constructing basic Lorentz covariants by using, besides  $p$ ,  $k_{F\perp}$  and  $n$ , the Lorentz scalar  $S_{LL}$ , Lorentz vector  $S_{LT}$ , and Lorentz tensor  $S_{TT}$ . We see that there are totally 40 tensor polarization dependent TMD FFs, 10 contribute at twist-2, 20 at twist-3 and the other 10 at twist-4. Among them, 24 (those related to  $\tilde{\Xi}_\alpha^{T(0)}$  and  $\Xi_{\rho\alpha}^{T(0)}$ ) are T-odd and the other 16 are T-even.

We emphasize in particular the similarities between the tensor polarization dependent terms given by Eqs. (A21-A25) in Appendix A and those unpolarized and vector polarization dependent terms given by Eqs. (A1-A14) in Appendix A.

(1) Since  $S_{LL}$  is a Lorentz scalar thus has no influence on the basic Lorentz covariants, the  $S_{LL}$ -dependent terms have exactly one to one correspondence to the unpolarized terms.

(2) For the  $S_{LT}$ -dependent terms, because  $S_{LT}$  and  $S$  behave differently under space reflection, the  $S_{LT}$ -dependent terms are different from the  $S$ -dependent terms. Since  $S_{LT}$  has only two independent transverse components, we have one to one correspondence for  $S_{LT}$ - to  $S_T$ -terms with the replacement of  $S_{T\alpha}$  by  $\tilde{S}_{LT\alpha}$ .

(3) Although there is no counterpart for the  $S_{TT}$ -dependent terms in other cases, however, there is no direct  $S_{TT\rho\alpha}$ -term contributing because  $S_{TT\rho\alpha} = S_{TT\alpha\rho}$  is symmetric while  $\Xi_{\rho\alpha}^{T(0)} = -\Xi_{\alpha\rho}^{T(0)}$  is anti-symmetric. All the independent  $S_{TT}$ -terms are in the form of  $S_{TT\alpha\sigma}k_{F\perp}^\sigma$  which is denoted by  $S_{TT\alpha}^{k_F}$ . Because  $S_{TT\alpha}^{k_F}$  has exactly the same Lorentz and space reflection behaviors as  $S_{LT\alpha}$ , we obtain a direct one to one correspondence between  $S_{LT}$ - and  $S_{TT}$ -dependent terms with the

replacement of  $S_{LT\alpha}$  by  $S_{TT\alpha}^{k_F}$ .

We note again the induced polarizations in the fragmentation of an unpolarized quark. We see that at leading twist an  $S_{LL}$ -dependent term exist and is described by  $D_{1LL}$ . There exist also terms depend on the other components of the tensor polarization at higher twists. We emphasize that, since they are independent of the polarization of the fragmenting quark, they might be much easier to study in experiments since no polarization in the initial state is needed.

We integrate over  $d^2k_{F\perp}$  and obtain Eqs. (A26-A30) in Appendix A. We have totally 8 terms, 4 of them are  $S_{LL}$ -dependent and the other 4 are  $S_{LT}$ -dependent. They have exact one to one correspondence to the unpolarized and  $S_T$ -dependent parts. We see that there is completely no  $S_{TT}$ -dependent terms exist in the one-dimensional case. This means that no  $S_{TT}$ -dependent one-dimensional FF can be defined via quark-quark correlator. The  $S_{TT}$ -dependent one-dimensional FFs can only be higher twists.

We list those twist-2 FFs in table II, and those twist-3 FFs in table III. The twist-4 FFs have the same structure of those at twist-2, so we will not make a separate table for them. We also list them according to chiral and time-reversal properties in table IV.

We note in particular the  $S_{LL}$ -dependent terms exist also in the one-dimensional case. We see that the leading twist contribution  $D_{1LL}$ -term survives the integration over  $k_{F\perp}$  also the higher twist addenda such as  $E_{LL}$  and  $D_{3LL}$ . This means that it can be studied even in inclusive high energy reactions. In the case that the leading twist effect dominates, the results should be not very much dependent of energy. The energy dependence can be used as a sensitive test of higher twist contributions.

### B. Relation to those defined via quark-gluon-quark correlator at twist-3

Higher twist PDFs and FFs can also be defined via the corresponding quark- $j$ -gluon-quark correlators ( $j = 1, 2, \dots$  represents the number of gluons) too [6–17]. However, because of QCD equation of motion  $\gamma \cdot D(y)\psi(y) = 0$ , the higher twist PDFs and FFs defined via these quark- $j$ -gluon-quark correlators are often not independent. They are related to those defined via the quark-quark correlator by a set of equations derived using the equation of motion and can often be replaced by using these relationships when calculating the cross sections and other measurable quantities for different high energy reactions. In this section, we take twist-3 as an example to illustrate the results for FFs defined via quark- $j$ -gluon-quark correlator and their relationships to those defined via quark-quark correlator.

Up to twist-3, we need to consider the quark-gluon-quark correlator defined as,

$$\begin{aligned} \hat{\Xi}_{\rho,ij}^{(1)}(k_F; p, S) = & \frac{1}{2\pi} \sum_X \int d^4\xi e^{-ik_F\xi} \langle p, S; X | \bar{\psi}_j(\xi) \mathcal{L}(\xi; \infty) | 0 \rangle \\ & \times \langle 0 | \mathcal{L}^\dagger(0; \infty) D_\rho(0) \psi_i(0) | p, S; X \rangle, \end{aligned} \quad (2.14)$$



where  $D_\rho(y) \equiv -i\partial_\rho + gA_\rho(y)$  and  $A_\rho(y)$  denotes the gluon field. Similar to the quark-quark correlator  $\hat{\Xi}^{(0)}$ , we decompose it as,

$$\begin{aligned} \hat{\Xi}_\rho^{(1)}(z, k_{F\perp}; p, S) &= \Xi_\rho^{(1)}(z, k_{F\perp}; p, S) + i\gamma_5 \tilde{\Xi}_\rho^{(1)}(z, k_{F\perp}; p, S) \\ &+ \gamma^\alpha \Xi_{\rho\alpha}^{(1)}(z, k_{F\perp}; p, S) + \gamma_5 \gamma^\alpha \tilde{\Xi}_{\rho\alpha}^{(1)}(z, k_{F\perp}; p, S) \\ &+ i\sigma^{\alpha\beta} \gamma_5 \Xi_{\rho\alpha\beta}^{(1)}(z, k_{F\perp}; p, S). \end{aligned} \quad (2.15)$$

Twist-3 components are the leading twist contributions that we obtain from  $\hat{\Xi}_\rho^{(1)}$ . There has to be one  $\bar{n}$  involved in the basic Lorentz covariants and the other(s) are from the transverse components. Since the  $\bar{n}$  component of gluon field goes into the gauge link, we only have the other three components for  $D_\rho$  thus no  $\bar{n}_\rho$ -component exists in the basic Lorentz covariants. We therefore do not have twist-3 contributions from  $\Xi_\rho^{(1)}$  or  $\tilde{\Xi}_\rho^{(1)}$ . The twist-3 contributions are obtained from  $\Xi_{\rho\alpha}^{(1)}$ ,  $\tilde{\Xi}_{\rho\alpha}^{(1)}$  and  $\Xi_{\rho\alpha\beta}^{(1)}$  and are given by Eqs. (A31-A39) in Appendix A. Here, we use a subscript  $d$  to specify that they are defined via quark-gluon-quark correlator. A prime in the superscript before the  $\perp$  denotes different polarization situation, that after the  $\perp$  specifies different FFs for the same polarization situation. We see that we have totally 36 FFs at twist-3 defined via quark-gluon-quark correlator. This is just the same as what we obtained from the quark-quark correlator. Among them, 18 are  $\chi$ -even and the other 18 are  $\chi$ -odd; 4 contribute to unpolarized part, 12 to vector polarized part and 20 to the tensor polarized part. We note in particular that the hermiticity in this case does not demand that the FFs defined via quark-gluon-quark correlator are real. They can have both real and imaginary parts.

For the 18 chiral even FFs (the  $D_d$ 's and  $G_d$ 's), QCD equation of motion leads to rather simple relationships. They can be written in the following unified form, i.e.,

$$D_{dS}^K(z, k_\perp) + G_{dS}^K(z, k_\perp) = \frac{1}{z} \left[ D_S^K(z, k_\perp) + iG_S^K(z, k_\perp) \right], \quad (2.16)$$

where the superscript  $K$  can be null (no superscript), a " $\perp$ " or a " $\perp\perp$ "; the subscript  $S$  specifies the polarization of hadron and can be null (unpolarized),  $L$ ,  $T$ ,  $LL$ ,  $LT$  or  $TT$ . There are in fact totally 9 such equations with the following combinations of  $K$  and  $S$ :  $K = \text{null}$  and  $S = T$  or  $LT$ ;  $K = \perp$  and  $S = \text{null}$ ,  $L$ ,  $T$ ,  $LL$ ,  $LT$  or  $TT$ ;  $K = \perp\perp$  and  $S = TT$ . For the 18 chiral odd FFs, we have also 9 equations in the form,

$$H_{dS}^K(z, k_\perp) + \frac{k_\perp^2}{2M^2} H_{dS}^{K'}(z, k_\perp) = \frac{1}{2z} \left[ H_S^K(z, k_\perp) + \frac{i}{2} E_S^K(z, k_\perp) \right], \quad (2.17)$$

with the following combinations of  $K$ ,  $K'$  and  $S$ :  $(K, K') = (\text{null}, \perp)$  and  $S = \text{null}$ ,  $L$  or  $LL$ ;  $(K, K') = (\perp, \perp)$  or  $(\perp\perp, \perp\perp)$  and  $S = T$ ,  $LT$ , or  $TT$ . We note in particular that these 18 equations in fact represent 36 real equations which imply that all the 36 twist-3 FFs defined via quark-quark correlator are given either by the real or imaginary part of those defined via quark-gluon-quark correlator. We note also that there are of course different choices for the basic Lorentz covariants used here in defining these FFs via quark-quark and/or quark-gluon-quark correlators. We choose them

in the way so the defined FFs satisfy the relationships given by Eqs. (2.16) and (2.17).

These relationships reveal the physical essences of these FFs and also help us to choose correct conventions in defining FFs. It is also very interesting to observe that, although not generally proved, the final results obtained for the physical observables up to twist-3 are all expressed in terms of FFs defined via quark-quark correlator [6–17]. The contributions from the quark-gluon-quark correlator can be replaced by using the relations given by Eqs. (2.16) and (2.17).

### III. KINEMATIC ANALYSIS OF $e^+e^- \rightarrow V\pi X$

As mentioned in the introduction, among all different high energy reactions,  $e^+e^-$ -annihilation is most suitable for studying FFs. For one-dimensional FFs, the inclusive hadron production process  $e^+e^- \rightarrow VX$  is the simplest case to study. In order to study transverse momentum dependence, we need at least two hadrons in the final state. Hence  $e^+e^- \rightarrow V\pi X$  as illustrated in Fig. 1 is most suitable for studying tensor polarization dependent part of the three dimensional FFs. We now concentrate on this reaction and present the results for cross sections in this and next sections.

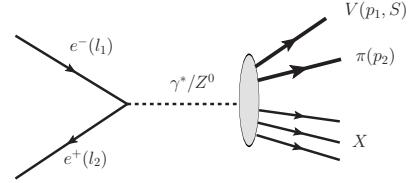


FIG. 1: Illustrating diagram for  $e^+e^- \rightarrow V\pi X$ .

For explicitness, we take  $e^+e^- \rightarrow Z^0 \rightarrow V\pi X$  as an example. The differential cross section is given by,

$$\frac{2E_1 E_2 d\sigma}{d^3 p_1 d^3 p_2} = \frac{\alpha^2 \chi}{s Q^4} L_{\mu\nu}(l_1, l_2) W^{\mu\nu}(q, p_1, S, p_2). \quad (3.1)$$

Here we use the same notations as illustrated in Fig. 1;  $\alpha = e^2/4\pi$ ,  $\chi = Q^4/[(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W$ ,  $Q^2 = s = q^2$ ,  $\theta_W$  is the Weinberg angle,  $M_Z$  is the Z-boson's mass and  $\Gamma_Z$  is the decay width. The leptonic tensor is well known and is given by,

$$L_{\mu\nu}(l_1, l_2) = c_1^e [l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - (l_1 \cdot l_2) g_{\mu\nu}] + i c_3^e \epsilon_{\mu\nu\rho\sigma} l_1^\rho l_2^\sigma, \quad (3.2)$$

where  $c_1^e = (c_V^e)^2 + (c_A^e)^2$  and  $c_3^e = 2c_V^e c_A^e$ ;  $c_V^e$  and  $c_A^e$  are defined in the weak interaction current  $\bar{\psi} \gamma^\mu (c_V^e - c_A^e \gamma^5) \psi$ . Similar notations are also used for quarks. The hadronic tensors is defined as,

$$\begin{aligned} W_{\mu\nu}(q, p_1, S, p_2) &= \frac{1}{(2\pi)^4} \sum_X (2\pi)^4 \delta^4(q - p_1 - p_2 - p_X) \\ &\times \langle 0 | J_\nu(0) | p_1, S, p_2, X \rangle \langle p_1, S, p_2, X | J_\mu(0) | 0 \rangle, \end{aligned} \quad (3.3)$$

where  $S$  denotes the polarization of the hadron and for vector meson it includes both the vector and tensor polarization parts,  $J_\mu(x) = \bar{\psi}(x)\Gamma_\mu\psi(x)$  and  $\Gamma_\mu = \gamma^\mu(c_V^q - c_A^q\gamma^5)$ .

Besides the Lorentz covariance, the hadronic tensor  $W^{\mu\nu}$  satisfies the general constraints imposed by Hermiticity, current conservation, and parity conservation in electromagnetic process, i.e.,

$$W^{*\mu\nu}(q, p_1, S, p_2) = W^{\nu\mu}(q, p_1, S, p_2), \quad (3.4)$$

$$q_\mu W^{\mu\nu}(q, p_1, S, p_2) = q_\nu W^{\mu\nu}(q, p_1, S, p_2) = 0, \quad (3.5)$$

$$W^{\mu\nu}(q, p_1, S, p_2) = W_{\mu\nu}(q^\mathcal{P}, p_1^\mathcal{P}, S^\mathcal{P}, p_2^\mathcal{P}). \quad (3.6)$$

We emphasize that parity conservation is not valid in weak process via  $Z$ -exchange.

### A. The general structure of $W^{\mu\nu}(q, p_1, S, p_2)$

A systematic analysis of the hadronic tensor  $W_{\mu\nu}$  for  $e^+e^- \rightarrow h_1 h_2 X$  for the case that both  $h_1$  and  $h_2$  are spin-1/2 hadrons are presented in [14]. Here, we extend the analysis to  $e^+e^- \rightarrow V\pi X$  including parity conserving as well as violating contributions. We present the results for the basic Lorentz tensors, the cross section and structure functions in the Lorentz invariant form as well as in the form of azimuthal angular dependences in a particular Lorentz frame.

#### 1. The basic Lorentz tensors for $W^{\mu\nu}(q, p_1, S, p_2)$

For the spin-independent and vector polarization dependent parts, the results can just be taken from [14]. We list them here for completeness and also for unification of notations that are more convenient to extend to including tensor polarization dependent parts.

First, the spin-independent (or unpolarized) part, we take the notations as,

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_{1q}^\mu p_{1q}^\nu, p_{1q}^{[\mu} p_{2q}^{\nu]}, p_{2q}^\mu p_{2q}^\nu \right\}, \quad (3.7)$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu q p_1 p_2\}}(p_{1q}, p_{2q})^{\nu\}} \right\}, \quad (3.8)$$

$$h_U^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]}, \quad (3.9)$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu q p_1}, \varepsilon^{\mu\nu q p_2} \right\}, \quad (3.10)$$

where  $h$  represents the parity conserved (space reflection  $P$ -even) tensors i.e. those satisfying Eq. (3.6) or more precisely  $h^{\mu\nu}(q^\mathcal{P}, p_1^\mathcal{P}, S^\mathcal{P}, p_2^\mathcal{P}) = h_{\mu\nu}(q, p_1, S, p_2)$  while  $\tilde{h}$  represents those parity non-conserved ( $P$ -odd) i.e. satisfying  $\tilde{h}^{\mu\nu}(q^\mathcal{P}, p_1^\mathcal{P}, S^\mathcal{P}, p_2^\mathcal{P}) = -\tilde{h}_{\mu\nu}(q, p_1, S, p_2)$ ; the superscript  $S$  or  $A$  denotes symmetric or anti-symmetric under exchange of  $(\mu \leftrightarrow \nu)$ , the subscript  $U$  denotes the unpolarized part [24]. A 4-momentum  $p$  with a subscript  $q$  denotes  $p_q \equiv p - q(p \cdot q)/q^2$  satisfying  $p_q \cdot q = 0$ . We use the short-handed notations to make the expressions more concise such as  $\varepsilon^{\mu q p_1 p_2} \equiv \varepsilon^{\mu\alpha\rho\sigma} q_\alpha p_{1\rho} p_{2\sigma}$ , and  $\varepsilon^{\{\mu q p_1 p_2\}}(p_{1q}, p_{2q})^{\nu\}}$  means  $\varepsilon^{\{\mu q p_1 p_2\}} p_{1q}^{\nu\}}$  and

$\varepsilon^{\{\mu q p_1 p_2\}} p_{2q}^{\nu\}}$ . We see that there are totally 9 such basic tensors in the unpolarized case.

For the vector polarization dependent part, we have,

$$h_{Vi}^{S\mu\nu} = \left\{ [(q \cdot S), (p_2 \cdot S)] \tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{S q p_1 p_2} h_{Uj}^{S\mu\nu} \right\}, \quad (3.11)$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \left\{ [(q \cdot S), (p_2 \cdot S)] h_{Ui}^{S\mu\nu}, \varepsilon^{S q p_1 p_2} \tilde{h}_{Uj}^{S\mu\nu} \right\}, \quad (3.12)$$

$$h_{Vi}^{A\mu\nu} = \left\{ [(q \cdot S), (p_2 \cdot S)] \tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{S q p_1 p_2} h_U^{A\mu\nu} \right\}, \quad (3.13)$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \left\{ [(q \cdot S), (p_2 \cdot S)] h_U^{A\mu\nu}, \varepsilon^{S q p_1 p_2} \tilde{h}_{Uj}^{A\mu\nu} \right\}. \quad (3.14)$$

There are totally 27 such  $S$ -dependent basic tensors, 3 times as many as those for the unpolarized part, corresponding to 3 independent vector polarization modes.

For the tensor polarization dependent part, after some lengthy algebra, we find out that if we consider  $S_{LL}$ -,  $S_{LT}$ - and  $S_{TT}$ -dependent parts separately, we obtained the following nice symmetric forms.

(1) *The  $S_{LL}$ -dependent part.* Since  $S_{LL}$  is a scalar, the  $S_{LL}$ -dependent part is very simple. The  $S_{LL}$ -dependent basic tensors are just given by the corresponding spin-independent tensors multiplied by  $S_{LL}$  such as  $h_{LLi}^{S\mu\nu} = S_{LL} h_{Ui}^{S\mu\nu}$  and so on. We have therefore 9 such tensors in this case.

(2) *The  $S_{LT}$ -dependent part.* In contrast to the axial-vector  $S$ ,  $S_{LT}$  is a vector satisfying the constraint  $S_{LT} \cdot p_1 = 0$ , the  $S_{LT}$ -dependent part is thus different from the  $S$ -dependent part. Furthermore, both  $S_{LT}$  and  $S_{TT}$  each has only two independent transverse components in the rest frame of the vector meson, this is guaranteed by demanding a further constraint  $S_{LT} \cdot q = 0$  for  $S_{LT}$ . The basic  $S_{LT}$ -dependent Lorentz tensors are given by,

$$h_{LTi}^{S\mu\nu} = \left\{ (p_2 \cdot S_{LT}) h_{Ui}^{S\mu\nu}, \varepsilon^{S_{LT} q p_1 p_2} \tilde{h}_{Uj}^{S\mu\nu} \right\}, \quad (3.15)$$

$$\tilde{h}_{LTi}^{S\mu\nu} = \left\{ (p_2 \cdot S_{LT}) \tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{S_{LT} q p_1 p_2} h_{Uj}^{S\mu\nu} \right\}, \quad (3.16)$$

$$h_{LTi}^{A\mu\nu} = \left\{ (p_2 \cdot S_{LT}) h_U^{A\mu\nu}, \varepsilon^{S_{LT} q p_1 p_2} \tilde{h}_{Uj}^{A\mu\nu} \right\}, \quad (3.17)$$

$$\tilde{h}_{LTi}^{A\mu\nu} = \left\{ (p_2 \cdot S_{LT}) \tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{S_{LT} q p_1 p_2} h_U^{A\mu\nu} \right\}. \quad (3.18)$$

There are totally 18 such tensors, corresponding to the two independent  $S_{LT}$ -components.

(3) *The  $S_{TT}$ -dependent part.*  $S_{TT}^{\alpha\beta}$  is a tensor satisfying the constraints,  $S_{TT}^{\alpha\beta} = S_{TT}^{\beta\alpha}$ ,  $g_{\alpha\beta} S_{TT}^{\alpha\beta} = 0$ ,  $S_{TT}^{p_1\beta} (\equiv S_{TT}^{\alpha\beta} p_{1\alpha}) = 0$  and  $S_{TT}^{q\beta} = 0$ . We have, the  $S_{TT}$ -dependent basic Lorentz tensors as given by,

$$h_{TTi}^{S\mu\nu} = \left\{ S_{TT}^{p_2 p_2} h_{Ui}^{S\mu\nu}, \varepsilon^{S_{TT}^2 q p_1 p_2} \tilde{h}_{Uj}^{S\mu\nu} \right\}, \quad (3.19)$$

$$\tilde{h}_{TTi}^{S\mu\nu} = \left\{ S_{TT}^{p_2 p_2} \tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{S_{TT}^2 q p_1 p_2} h_{Uj}^{S\mu\nu} \right\}, \quad (3.20)$$

$$h_{TTi}^{A\mu\nu} = \left\{ S_{TT}^{p_2 p_2} h_U^{A\mu\nu}, \varepsilon^{S_{TT}^2 q p_1 p_2} \tilde{h}_{Uj}^{A\mu\nu} \right\}, \quad (3.21)$$

$$\tilde{h}_{TTi}^{A\mu\nu} = \left\{ S_{TT}^{p_2 p_2} \tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{S_{TT}^2 q p_1 p_2} h_U^{A\mu\nu} \right\}. \quad (3.22)$$

There are also totally 18  $S_{TT}$ -dependent basic Lorentz tensors. For  $W_{\mu\nu}(q, p_1, S, p_2)$ , we have totally 81 basic Lorentz tensors, 41 of them are space reflection even and 40 are odd.

## 2. General form of $W^{\mu\nu}(q, p_1, S, p_2)$

The hadronic tensor  $W^{\mu\nu}(q, p_1, S, p_2)$  is in general expressed as a sum of all these basic Lorentz tensors multiplied by corresponding coefficients. The coefficients are real and functions of the Lorentz scalars  $q^2$ ,  $q \cdot p_1$ ,  $q \cdot p_2$  and  $p_1 \cdot p_2$ , which can be replaced by  $s = q^2$ ,  $\xi_1 = 2q \cdot p_1/q^2$ ,  $\xi_2 = 2q \cdot p_2/q^2$  and  $\xi_{12} = s_{12}/s = (p_1 + p_2)^2/s$ . More precisely, we have,

$$W^{\mu\nu}(q, p_1, S, p_2) = W^{S\mu\nu}(q, p_1, S, p_2) + iW^{A\mu\nu}(q, p_1, S, p_2), \quad (3.23)$$

$$W^{S\mu\nu}(q, p_1, S, p_2) = \sum_{\sigma,i} W_{\sigma i}^S(s, \xi_1, \xi_2, \xi_{12}) h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,j} \tilde{W}_{\sigma j}^S(s, \xi_1, \xi_2, \xi_{12}) \tilde{h}_{\sigma j}^{S\mu\nu}, \quad (3.24)$$

$$W^{A\mu\nu}(q, p_1, S, p_2) = \sum_{\sigma,i} W_{\sigma i}^A(s, \xi_1, \xi_2, \xi_{12}) h_{\sigma i}^{A\mu\nu} + \sum_{\sigma,j} \tilde{W}_{\sigma j}^A(s, \xi_1, \xi_2, \xi_{12}) \tilde{h}_{\sigma j}^{A\mu\nu}, \quad (3.25)$$

where the subscript  $\sigma$  denotes  $U, V, LL, LT$  and  $TT$  for different polarizations; all the coefficients  $W$ 's are scalar functions of the Lorentz scalars  $s, \xi_1, \xi_2$  and  $\xi_{12}$ .

### B. The general structure for the cross section

Since the number of independent structure functions is rather large, in practice, it is often more convenient to write the cross section directly.

#### 1. The Lorentz invariant form

Making the Lorentz contraction of  $W^{\mu\nu}(q, p_1, S, p_2)$  with  $L_{\mu\nu}(l_1, l_2)$ , we obtain the general form of the cross section. For the unpolarized part, this is given by,

$$\frac{2E_1 E_2 d\sigma^U}{d^3 p_1 d^3 p_2} = \frac{\alpha^2 \chi}{s^2} [\mathcal{F}_U(s, \xi_1, \xi_2, \xi_{12}; y_1, y_2) + \tilde{\mathcal{F}}_U(s, \xi_1, \xi_2, \xi_{12}; y_1, y_2, \tilde{y})], \quad (3.26)$$

where  $\mathcal{F}_U$  and  $\tilde{\mathcal{F}}_U$  represent the space reflection even and odd parts respectively and they have the structures as given by,

$$\mathcal{F}_U = F_U^0 + F_U^1 y_1 + F_U^2 y_2 + F_U^{11} y_1^2 + F_U^{22} y_2^2 + F_U^{12} y_1 y_2, \quad (3.27)$$

$$\tilde{\mathcal{F}}_U = \tilde{y}(\tilde{F}_U^0 + \tilde{F}_U^1 y_1 + \tilde{F}_U^2 y_2), \quad (3.28)$$

where besides  $\xi_1, \xi_2$  and  $\xi_{12}$  defined before, we introduced two new Lorentz scalars  $y_1 = 2p_1 \cdot l_1/q^2$ ,  $y_2 = 2p_2 \cdot l_1/q^2$  and one pseudo-scalar  $\tilde{y} = \varepsilon^{l_1 q p_1 p_2}/q^4$ . The “structure functions”  $F$ 's are all scalar functions depending on  $(s, \xi_1, \xi_2, \xi_{12})$ . We see also clearly that the six  $F$ 's describe the parity conserved

contributions while the three  $\tilde{F}$ 's represent the parity violated part. They are related to the  $W$ 's by,

$$F_U^0 = -\frac{1}{2}c_1^e [2W_{U1}^S + (m_1^2 W_{U2}^S + m_2^2 W_{U3}^S) - (s\xi_{12} - m_1^2 - m_2^2)W_{U4}^S] + \frac{1}{2}sc_3^e(\xi_1 \tilde{W}_{U1}^A + \xi_2 \tilde{W}_{U2}^A), \quad (3.29)$$

$$F_U^1 = \frac{1}{2}c_1^e s(\xi_1 W_{U2}^S + \xi_2 W_{U4}^S) - c_3^e s \tilde{W}_{U1}^A, \quad (3.30)$$

$$F_U^2 = \frac{1}{2}c_1^e s(\xi_2 W_{U3}^S + \xi_1 W_{U4}^S) - c_3^e s \tilde{W}_{U2}^A, \quad (3.31)$$

$$F_U^{11} = -\frac{1}{2}c_1^e s W_{U2}^S, \quad (3.32)$$

$$F_U^{22} = -\frac{1}{2}c_1^e s W_{U3}^S, \quad (3.33)$$

$$F_U^{12} = -c_1^e s W_{U4}^S, \quad (3.34)$$

$$\tilde{F}_U^0 = c_1^e s^2(\xi_1 \tilde{W}_{U1}^S + \xi_2 \tilde{W}_{U2}^S) - 2c_3^e s W_U^A, \quad (3.35)$$

$$\tilde{F}_U^1 = -2c_1^e s^2 \tilde{W}_{U1}^S, \quad (3.36)$$

$$\tilde{F}_U^2 = -2c_1^e s^2 \tilde{W}_{U2}^S. \quad (3.37)$$

We see here that although the  $F_{Ui}$ 's and  $\tilde{F}_{Ui}$ 's are all functions of  $s, \xi_1, \xi_2, \xi_{12}$ , they contain already information from the leptonic tensor due to the coefficient  $c_1^e$  and  $c_3^e$ . We also see that the parity conserved parts come from parity conserved hadronic tensor terms (characterized by  $W$ 's) contracted with parity conserved leptonic tensor terms (characterized by  $c_1^e$ ) or parity violated hadronic tensor terms (characterized by  $\tilde{W}$ 's) contracted with the parity violated leptonic tensor term (characterized by  $c_3^e$ ). We have six such  $F_{Ui}$ 's. Similarly we have three  $\tilde{F}_{Ui}$ 's for the parity violated parts obtained from Lorentz contractions of parity conserved leptonic tensor terms with parity violated hadronic tensor terms or parity violated leptonic tensor term with parity conserved tensor terms.

The polarization dependent part has completely the same structure. For the vector polarization dependent part, from Eqs. (3.11-3.14), we obtain immediately that,

$$\frac{2E_1 E_2 d\sigma^V}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi \{ (q \cdot S)(\mathcal{F}_{V1} + \tilde{\mathcal{F}}_{V1}) + (p_2 \cdot S)(\mathcal{F}_{V2} + \tilde{\mathcal{F}}_{V2}) + \varepsilon^{S q p_1 p_2}(\mathcal{F}_{V3} + \tilde{\mathcal{F}}_{V3}) \}. \quad (3.38)$$

Here, we note that since  $q \cdot S$  and  $p_2 \cdot S$  are space reflection odd hence the parity conserved parts  $\mathcal{F}_{V1}$  and  $\mathcal{F}_{V2}$  take exactly the same form as  $\tilde{\mathcal{F}}_U$  given by Eq. (3.28), while the parity violated parts  $\tilde{\mathcal{F}}_{V1}$  and  $\tilde{\mathcal{F}}_{V2}$  take the same form as  $\mathcal{F}_U$  given by Eq. (3.27) with the subscript  $U$  replaced by  $V1$  or  $V2$ . Since  $\varepsilon^{S q p_1 p_2}$  is a scalar,  $\mathcal{F}_{V3}$  and  $\tilde{\mathcal{F}}_{V3}$  take exactly the same form as  $\mathcal{F}_U$  and  $\tilde{\mathcal{F}}_U$  given by Eqs. (3.27-3.37) respectively with the subscript  $U$  replaced by  $V3$ . We have three set of  $F_{Vi}$  and  $\tilde{F}_{Vi}$  because there are three independent components of vector polarization.

For the tensor polarization dependent part, we have,

$$\frac{2E_1 E_2 d\sigma^{LL}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL}), \quad (3.39)$$

$$\frac{2E_1 E_2 d\sigma^{LT}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi \{ (p_2 \cdot S_{LT})(\mathcal{F}_{LT1} + \tilde{\mathcal{F}}_{LT1})$$

$$+ \mathcal{E}^{S_{LT} q p_1 p_2} (\mathcal{F}_{LT2} + \tilde{\mathcal{F}}_{LT2}) \}, \quad (3.40)$$

$$\frac{2E_1 E_2 d\sigma^{TT}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi \left\{ S_{TT}^{p_2 p_2} (\mathcal{F}_{TT1} + \tilde{\mathcal{F}}_{TT1}) \right. \\ \left. + \mathcal{E}^{S_{TT}^{p_2} q p_1 p_2} (\mathcal{F}_{TT2} + \tilde{\mathcal{F}}_{TT2}) \right\}. \quad (3.41)$$

Here  $S_{LL}$ ,  $p_2 \cdot S_{LT}$  and  $S_{TT}^{p_2 p_2}$  are scalars,  $\mathcal{E}^{S_{LT} q p_1 p_2}$  and  $\mathcal{E}^{S_{TT}^{p_2} q p_1 p_2}$  are pseudo-scalars. Hence,  $\mathcal{F}_{LL}$ ,  $\mathcal{F}_{LT1}$ ,  $\mathcal{F}_{TT1}$ ,  $\tilde{\mathcal{F}}_{LT2}$  and  $\tilde{\mathcal{F}}_{TT2}$  take exactly the same form as  $\mathcal{F}_U$  given by Eq. (3.27), while  $\tilde{\mathcal{F}}_{LL}$ ,  $\tilde{\mathcal{F}}_{LT1}$ ,  $\tilde{\mathcal{F}}_{TT1}$ ,  $\mathcal{F}_{LT2}$  and  $\mathcal{F}_{TT2}$  take exactly the same form as  $\tilde{\mathcal{F}}_U$  given by Eq. (3.28).

## 2. In the Helicity-GJ-frame

Going into a special reference frame, we can express the cross section in terms of angular dependences. The polarization of high energy particles are described and/or studied most conveniently in the helicity frame, i.e. where we choose the direction of motion of the particle as  $z$ -direction. Hence, to study polarization dependent FFs for  $V$  in  $e^+ e^- \rightarrow V \pi X$ , we suggest to choose the following frame. We choose center of mass frame of the  $e^+ e^-$ -system, and direction of motion of  $V$  i.e.  $\vec{p}_1$  as  $z$ -direction, and the lepton-hadron (vector meson) plane as  $Oxz$  plane. This is a particular Gottfried-Jackson frame [26] which we will refer to as ‘‘Helicity-GJ frame’’ in the following of this paper. In this frame, we have,

$$p_1 = (E_1, 0, 0, p_{1z}), \quad (3.42)$$

$$p_2 = (E_2, |\vec{p}_{2T}| \cos \varphi, |\vec{p}_{2T}| \sin \varphi, p_{2z}), \quad (3.43)$$

$$l_1 = \frac{Q}{2} (1, \sin \theta, 0, \cos \theta), \quad (3.44)$$

$$l_2 = \frac{Q}{2} (1, -\sin \theta, 0, -\cos \theta), \quad (3.45)$$

$$q = l_1 + l_2 = (Q, 0, 0, 0), \quad (3.46)$$

and we choose  $\xi_1, \xi_2, |\vec{p}_{2T}|, \theta$  or  $y = l_2 \cdot p_1 / q \cdot p_1 \approx (1 + \cos \theta)/2$  and  $\varphi$  as the independent variable set. The other variables are replaced. The basic volume element transforms as,

$$\frac{d^3 p_1 d^3 p_2}{E_1 E_2} = \frac{\pi \xi_1}{\xi_2} s (1 - 4M_{2T}^2 / s \xi_2^2)^{-1/2} d\xi_1 d\xi_2 dy d^2 p_{2T}, \quad (3.47)$$

where  $M_{2T}^2 = M_2^2 + \vec{p}_{2T}^2$  and  $d^2 p_{2T} = d\vec{p}_{2T}^2 d\varphi/2$ .

### (i) The structure functions

For the unpolarized part, we have,

$$\mathcal{F}_U = (1 + \cos^2 \theta) F_{1U} + \sin^2 \theta F_{2U} + \cos \theta F_{3U} \\ + \cos \varphi [\sin \theta F_{1U}^{\cos \varphi} + \sin 2\theta F_{2U}^{\cos \varphi}] \\ + \cos 2\varphi \sin^2 \theta F_U^{\cos 2\varphi}, \quad (3.48)$$

$$\tilde{\mathcal{F}}_U = \sin \varphi [\sin \theta \tilde{F}_{1U}^{\sin \varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin \varphi}] \\ + \sin 2\varphi \sin^2 \theta \tilde{F}_U^{\sin 2\varphi}, \quad (3.49)$$

where  $F_{Ui}$  and  $\tilde{F}_{Ui}$  are all scalar functions of  $s, \xi_1, \xi_2$  and  $p_{2T}^2$ . We see also clearly that we have totally 9 independent structure functions in the unpolarized case, 6 of them are denoted

by  $F_U$ 's and correspond to parity conserving terms and the other 3 are  $\tilde{F}_U$ 's describing parity odd part of the cross section. This is just the same as those shown by Eqs. (3.29-3.37). We note in particular that the structure functions  $F_U$ 's and  $\tilde{F}_U$ 's themselves are scalar functions of  $s, \xi_1, \xi_2$  and  $p_{2T}^2$  and are invariant under space reflection. But the angular dependent coefficients have the corresponding space reflection properties. The different basic Lorentz tensors  $h_{Ui}^{\mu\nu}$ 's and  $\tilde{h}_{Ui}^{\mu\nu}$ 's are transformed to different angular dependences. We also see that there are 3 azimuthal angle independent structure functions, 3 parity conserving and 3 parity violating azimuthal angle dependent structure functions. They correspond to  $\cos$  or  $\sin$  asymmetries and are parity conserving and violating respectively.

Here we take the following conventions for the notations of structure functions, i.e., the superscript to denote the corresponding azimuthal angle  $\varphi$ -dependence, the capital letter in the subscripts to denote the polarization, the digital number in front of the capital letter to specify if we have more than one such structure functions corresponding to the same azimuthal angle  $\varphi$ -dependence but different  $\theta$ - or  $y$ -dependences [25]. We also note that to replace  $\theta$  by  $y$  we have,

$$1 + \cos^2 \theta \approx 1 + (2y - 1)^2 = 2A(y), \quad (3.50)$$

$$\cos \theta \approx -1 + 2y = -B(y), \quad (3.51)$$

$$\sin^2 \theta \approx 1 - (1 - 2y)^2 = 4y(1 - y) = C(y), \quad (3.52)$$

that appear frequently in the expressions of the cross section.

For the vector polarized part, we note that,

$$S = (\lambda \frac{p_{1z}}{M_1}, |\vec{S}_T| \cos \varphi_S, |\vec{S}_T| \sin \varphi_S, \lambda \frac{E_1}{M_1}). \quad (3.53)$$

The  $(q \cdot S)$ - and  $\mathcal{E}^{S q p_1 p_2}$ -terms in Eq. (3.38) contribute to longitudinal and transverse polarization separately, while the  $(p_2 \cdot S)$ -terms contribute to both cases. The contributions to transverse polarization from  $(p_2 \cdot S)$ - and  $\mathcal{E}^{S q p_1 p_2}$ -terms are characterized by additional  $\cos(\varphi_S - \varphi)$ - and  $\sin(\varphi_S - \varphi)$ -dependence. We absorb the different kinematic factors into  $\mathcal{F}$  and  $\tilde{\mathcal{F}}$  and write the cross section as,

$$\frac{2E_1 E_2 d\sigma^V}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi \left\{ \lambda (\mathcal{F}_L + \tilde{\mathcal{F}}_L) + |\vec{S}_T| (\mathcal{F}_T + \tilde{\mathcal{F}}_T) \right\}. \quad (3.54)$$

Since  $\lambda$  changes sign under space reflection, the parity conserving  $\mathcal{F}_L$  and parity violating  $\tilde{\mathcal{F}}_L$  take exactly the same form as  $\mathcal{F}_U$  and  $\tilde{\mathcal{F}}_U$  respectively. We have 3  $F_{iL}$ 's that have one to one correspondence to  $\tilde{F}_{iU}$ 's and 6  $\tilde{F}_{iL}$ 's that have one to one correspondence to  $F_{iU}$ 's.

For the transverse (vector) polarization dependent part, due to  $\varphi_S$ -dependence, the structure looks a bit different, they are given by,

$$\mathcal{F}_T = \sin \varphi_S [\sin \theta F_{1T}^{\sin \varphi_S} + \sin 2\theta F_{2T}^{\sin \varphi_S}] \\ + \sin(\varphi_S + \varphi) \sin^2 \theta F_T^{\sin(\varphi_S + \varphi)} \\ + \sin(\varphi_S - \varphi) [(1 + \cos^2 \theta) F_{1T}^{\sin(\varphi_S - \varphi)} \\ + \sin^2 \theta F_{2T}^{\sin(\varphi_S - \varphi)} + \cos \theta F_{3T}^{\sin(\varphi_S - \varphi)}]$$



$$\begin{aligned}
& + \sin(\varphi_S - 2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}] \\
& + \sin(\varphi_S - 3\varphi) \sin^2 \theta F_T^{\sin(\varphi_S-3\varphi)}, \quad (3.55) \\
\tilde{\mathcal{F}}_T = & \cos\varphi_S [\sin\theta \tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos\varphi_S}] \\
& + \cos(\varphi_S + \varphi) \sin^2 \theta \tilde{F}_T^{\cos(\varphi_S+\varphi)} \\
& + \cos(\varphi_S - \varphi)[(1 + \cos^2 \theta) \tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} \\
& \quad + \sin^2 \theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}] \\
& + \cos(\varphi_S - 2\varphi)[\sin\theta \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}] \\
& + \cos(\varphi_S - 3\varphi) \sin^2 \theta \tilde{F}_T^{\cos(\varphi_S-3\varphi)}. \quad (3.56)
\end{aligned}$$

There are 18 such transverse polarization dependent structure functions, 9 of them are space reflection even and 9 are space reflection odd. Totally we have 27 vector polarization dependent structure functions corresponding to the 27 independent basic Lorentz tensors  $h_{V_i}^{\mu\nu}$ 's for the hadronic tensor. Among them, 12 contribute to space reflection even terms in the cross section, the other 15 to space reflection odd terms. We note in particular the  $\sin\varphi_S$ - and  $\cos\varphi_S$ -terms correspond to single transverse spin asymmetries in deep-inelastic lepton-nucleon scattering  $e^-h \rightarrow e^-X$  with respect to the leptonic plane. They are either parity or time reversal odd and do not exist in  $e^-h \rightarrow e^-X$ . In  $e^+e^-$ -annihilation, they describe the transverse polarization in or transverse to the lepton-hadron plane.

The  $S_{LL}$ -dependent part is again completely the same as that for the unpolarized case, i.e., we have a one to one correspondence of  $F_{LL}$  to  $F_U$  and  $\tilde{F}_{LL}$  to  $\tilde{F}_U$ .

For the  $S_{LT}$ -dependent part, we define,

$$S_{LT}^x = |S_{LT}| \cos\varphi_{LT}, \quad (3.57)$$

$$S_{LT}^y = |S_{LT}| \sin\varphi_{LT}, \quad (3.58)$$

$$|S_{LT}| = \sqrt{(S_{LT}^x)^2 + (S_{LT}^y)^2}, \quad (3.59)$$

and we have,

$$\frac{2E_1 E_2 d\sigma^{LT}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi |S_{LT}| \{\mathcal{F}_{LT} + \tilde{\mathcal{F}}_{LT}\}, \quad (3.60)$$

Because  $S_{LT}$  behaves differently from  $S_T$  under space reflection, we obtain that  $\mathcal{F}_{LT}$  takes exactly the same form as  $\tilde{\mathcal{F}}_T$  and  $\tilde{\mathcal{F}}_{LT}$  behaves in the same way as  $\mathcal{F}_T$ . More precisely, we obtain the results for  $\mathcal{F}_{LT}$  by replacing  $\varphi_S$  with  $\varphi_{LT}$  and  $\tilde{F}_{jT}$  with  $F_{jLT}$  in Eq. (3.56), and those for  $\tilde{\mathcal{F}}_{LT}$  by replacing  $\varphi_S$  with  $\varphi_{LT}$  and  $F_{jT}$  with  $\tilde{F}_{jLT}$  in Eq. (3.55). We have exactly one to one correspondence here.

For the  $S_{TT}$ -dependent part, we take,

$$S_{TT}^{xx} = |S_{TT}| \cos 2\varphi_{TT}, \quad (3.61)$$

$$S_{TT}^{xy} = |S_{TT}| \sin 2\varphi_{TT}, \quad (3.62)$$

$$|S_{TT}| = \sqrt{(S_{TT}^{xx})^2 + (S_{TT}^{xy})^2}, \quad (3.63)$$

so that  $S_{TT}^{p_2 p_2}$  and  $S_{TT}^{p_2 q p_1 p_2}$  will contribute  $\cos(2\varphi_{TT} - 2\varphi)$  and  $\sin(2\varphi_{TT} - 2\varphi)$  terms. Compare with the  $S_T$  part, by changing  $\varphi_S \rightarrow 2\varphi_{TT} - \varphi$ , the  $S_{TT}$ -dependent part is classified into  $\cos 2\varphi_{TT}$ -,  $\cos(2\varphi_{TT} - \varphi)$ -,  $\cos(2\varphi_{TT} - 2\varphi)$ -,  $\cos(2\varphi_{TT} - 3\varphi)$ -,

$\cos(2\varphi_{TT} - 4\varphi)$ - and the corresponding sin terms. More precisely, they are given by,

$$\frac{2E_1 E_2 d\sigma^{TT}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{s^2} \chi |S_{TT}| \{\mathcal{F}_{TT} + \tilde{\mathcal{F}}_{TT}\}, \quad (3.64)$$

$$\begin{aligned}
\mathcal{F}_{TT} = & \cos 2\varphi_{TT} \sin^2 \theta F_{TT}^{\cos 2\varphi_{TT}} \\
& + \cos(2\varphi_{TT} - \varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}] \\
& + \cos(2\varphi_{TT} - 2\varphi)[(1 + \cos^2 \theta) F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} \\
& \quad + \sin^2 \theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}] \\
& + \cos(2\varphi_{TT} - 3\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}] \\
& + \cos(2\varphi_{TT} - 4\varphi) \sin^2 \theta F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}, \quad (3.65)
\end{aligned}$$

$$\begin{aligned}
\tilde{\mathcal{F}}_{TT} = & \sin 2\varphi_{TT} \sin^2 \theta \tilde{F}_{TT}^{\sin 2\varphi_{TT}} \\
& + \sin(2\varphi_{TT} - \varphi)(\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-\varphi)} + \sin 2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-\varphi)}) \\
& + \sin(2\varphi_{TT} - 2\varphi)[(1 + \cos^2 \theta) \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} \\
& \quad + \sin^2 \theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}] \\
& + \sin(2\varphi_{TT} - 3\varphi)(\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-3\varphi)} + \sin 2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}) \\
& + \sin(2\varphi_{TT} - 4\varphi) \sin^2 \theta \tilde{F}_{TT}^{\sin(2\varphi_{TT}-4\varphi)}. \quad (3.66)
\end{aligned}$$

To show the regularities we list all the 81 structure functions together with the leading twist parton model results in a table. See table I in Sec. V.

### (ii) The azimuthal asymmetries

From these equations, we can calculate the azimuthal asymmetries and different components of hadron polarization in a straightforward way. E.g.,

$$\langle \cos\varphi \rangle_U = (\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi})/2F_{U_t}, \quad (3.67)$$

$$\langle \cos 2\varphi \rangle_U = \sin^2 \theta F_U^{\cos 2\varphi}/2F_{U_t}, \quad (3.68)$$

$$\langle \sin\varphi \rangle_U = (\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi})/2F_{U_t}, \quad (3.69)$$

$$\langle \sin 2\varphi \rangle_U = \sin^2 \theta \tilde{F}_U^{\sin 2\varphi}/2F_{U_t}, \quad (3.70)$$

where  $F_{U_t}$  denotes the result of  $\mathcal{F}_U + \tilde{\mathcal{F}}_U$  averaging over  $\varphi$ , i.e.,

$$\begin{aligned}
F_{U_t}(s, \xi_1, \xi_2, p_{2T}, \theta) & \equiv \int \frac{d\varphi}{2\pi} (\mathcal{F}_U + \tilde{\mathcal{F}}_U) \\
& = (1 + \cos^2 \theta) F_{1U} + \sin^2 \theta F_{2U} + \cos\theta F_{3U}. \quad (3.71)
\end{aligned}$$

We see that these azimuthal asymmetries just equal to the corresponding structure functions divided by the azimuthal angle independent part. We also see that the cos-asymmetries correspond to parity conserving part and the sin-asymmetries correspond to parity violating part of the cross section so the latter vanish in parity conserving processes.

### (iii) The polarization of the vector meson V

The average value of each component of the polarization is obtained from their correspondences to the probability differences in different polarization such as  $\tilde{S}_{LL} = [1 -$

$3\mathcal{P}(0; 0, 0)]/2$  where  $\mathcal{P}(m; \theta_n, \phi_n)$  is the probability for  $V$  to be in the eigenstate of  $\Sigma^n$  with the eigenvalue  $m$  [21]. For the five components describing the tensor polarization, we obtain,

$$\bar{S}_{LL} = \frac{1}{2} \frac{\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL}}{\mathcal{F}_U + \tilde{\mathcal{F}}_U}, \quad (3.72)$$

$$\bar{S}_{LT}^i = \frac{2}{3} \frac{\mathcal{F}_{LT}^i + \tilde{\mathcal{F}}_{LT}^i}{\mathcal{F}_U + \tilde{\mathcal{F}}_U}, \quad (3.73)$$

$$\bar{S}_{TT}^{xi} = \frac{2}{3} \frac{\mathcal{F}_{TT}^{xi} + \tilde{\mathcal{F}}_{TT}^{xi}}{\mathcal{F}_U + \tilde{\mathcal{F}}_U}, \quad (3.74)$$

where  $i = x$  or  $y$  denotes different components of the polarization tensor. It is also interesting to see that the numerator  $\mathcal{F}_{LT}^x$  and  $\mathcal{F}_{LT}^y$  are equal to the  $\cos \varphi_{LT}$  and  $\sin \varphi_{LT}$ -terms of  $\mathcal{F}_{LT}$  respectively. They can be obtained as follows,

$$\mathcal{F}_{LT}^x = \int \frac{d\varphi_{LT}}{\pi} \cos \varphi_{LT} \mathcal{F}_{LT}, \quad (3.75)$$

$$\mathcal{F}_{LT}^y = \int \frac{d\varphi_{LT}}{\pi} \sin \varphi_{LT} \mathcal{F}_{LT}, \quad (3.76)$$

and similar for  $\tilde{\mathcal{F}}_{LT}^i$ . For  $\mathcal{F}_{TT}^{xi}$ , we have,

$$\mathcal{F}_{TT}^{xx} = \int \frac{d\varphi_{TT}}{\pi} \cos 2\varphi_{TT} \mathcal{F}_{TT}, \quad (3.77)$$

$$\mathcal{F}_{TT}^{xy} = \int \frac{d\varphi_{TT}}{\pi} \sin 2\varphi_{TT} \mathcal{F}_{TT}, \quad (3.78)$$

and similar for  $\tilde{\mathcal{F}}_{TT}^{xi}$ . The explicit expressions can be obtained easily from those for the corresponding  $\mathcal{F}_\sigma$  or  $\tilde{\mathcal{F}}_\sigma$ . We omit them here but simply emphasize that they are in general dependent on the variables  $s, \xi_1, \xi_2, p_{2T}, \theta$  and  $\varphi$ .

If we average over  $\varphi$ , we see that only the  $\varphi$  independent terms in the expressions of  $\mathcal{F}$ 's and  $\tilde{\mathcal{F}}$ 's survive. We denote them as,

$$\langle \mathcal{F}_\sigma \rangle = \int \frac{d\varphi}{2\pi} \mathcal{F}_\sigma, \quad (3.79)$$

and we obtain,

$$\langle \mathcal{F}_U \rangle = (1 + \cos^2 \theta) F_{1U} + \sin^2 \theta F_{2U} + \cos \theta F_{3U}, \quad (3.80)$$

$$\langle \tilde{\mathcal{F}}_U \rangle = 0, \quad (3.81)$$

$$\langle \mathcal{F}_L \rangle = 0, \quad (3.82)$$

$$\langle \tilde{\mathcal{F}}_L \rangle = (1 + \cos^2 \theta) \tilde{F}_{1L} + \sin^2 \theta \tilde{F}_{2L} + \cos \theta \tilde{F}_{3L}, \quad (3.83)$$

$$\langle \mathcal{F}_T \rangle = \sin \varphi_S \left( \sin \theta F_{1T}^{\sin \varphi_S} + \sin 2\theta F_{2T}^{\sin \varphi_S} \right), \quad (3.84)$$

$$\langle \tilde{\mathcal{F}}_T \rangle = \cos \varphi_S \left( \sin \theta \tilde{F}_{1T}^{\cos \varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos \varphi_S} \right), \quad (3.85)$$

$$\langle \mathcal{F}_{LT} \rangle = \cos \varphi_{LT} \left( \sin \theta F_{1LT}^{\cos \varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos \varphi_{LT}} \right), \quad (3.86)$$

$$\langle \tilde{\mathcal{F}}_{LT} \rangle = \sin \varphi_{LT} \left( \sin \theta \tilde{F}_{1LT}^{\sin \varphi_{LT}} + \sin 2\theta \tilde{F}_{2LT}^{\sin \varphi_{LT}} \right), \quad (3.87)$$

$$\langle \mathcal{F}_{LL} \rangle = (1 + \cos^2 \theta) F_{1LL} + \sin^2 \theta F_{2LL} + \cos \theta F_{3LL}, \quad (3.88)$$

$$\langle \tilde{\mathcal{F}}_{LL} \rangle = 0, \quad (3.89)$$

$$\langle \mathcal{F}_{TT} \rangle = \cos 2\varphi_{TT} \sin^2 \theta F_{TT}^{\cos 2\varphi_{TT}}, \quad (3.90)$$

$$\langle \tilde{\mathcal{F}}_{TT} \rangle = \sin 2\varphi_{TT} \sin^2 \theta \tilde{F}_{TT}^{\sin 2\varphi_{TT}}. \quad (3.91)$$

We see the similarities between different components and also the  $\cos \varphi_\sigma$  or  $\sin \varphi_\sigma$ -term corresponding to  $x$  or  $y$ -component of the polarization. More precisely, in this case, we obtain,

$$\langle \lambda \rangle = \frac{2}{3F_{U_t}} \left( (1 + \cos^2 \theta) \tilde{F}_{1L} + \sin^2 \theta \tilde{F}_{2L} + \cos \theta \tilde{F}_{3L} \right), \quad (3.92)$$

$$\langle S_{LL} \rangle = \frac{1}{2F_{U_t}} \left( (1 + \cos^2 \theta) F_{1LL} + \sin^2 \theta F_{2LL} + \cos \theta F_{3LL} \right), \quad (3.93)$$

$$\langle S_T^x \rangle = \frac{2}{3F_{U_t}} \left( \sin \theta \tilde{F}_{1T}^{\cos \varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos \varphi_S} \right), \quad (3.94)$$

$$\langle S_T^y \rangle = \frac{2}{3F_{U_t}} \left( \sin \theta F_{1T}^{\sin \varphi_S} + \sin 2\theta F_{2T}^{\sin \varphi_S} \right), \quad (3.95)$$

$$\langle S_{LT}^x \rangle = \frac{2}{3F_{U_t}} \left( \sin \theta F_{1LT}^{\cos \varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos \varphi_{LT}} \right), \quad (3.96)$$

$$\langle S_{LT}^y \rangle = \frac{2}{3F_{U_t}} \left( \sin \theta \tilde{F}_{1LT}^{\sin \varphi_{LT}} + \sin 2\theta \tilde{F}_{2LT}^{\sin \varphi_{LT}} \right), \quad (3.97)$$

$$\langle S_{TT}^{xx} \rangle = \frac{2}{3F_{U_t}} \sin^2 \theta F_{TT}^{\cos 2\varphi_{TT}}, \quad (3.98)$$

$$\langle S_{TT}^{xy} \rangle = \frac{2}{3F_{U_t}} \sin^2 \theta \tilde{F}_{TT}^{\sin 2\varphi_{TT}}. \quad (3.99)$$

We see that in this way we just pick up the corresponding  $\varphi$ -independent and, in the transverse polarization case, the  $\cos \varphi_\sigma$  or  $\sin \varphi_\sigma$ -terms. These results are much simpler and can be used to study the corresponding components of the structure functions more conveniently. We also note that  $\langle S_{LL} \rangle$ ,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^x \rangle$  and  $\langle S_{TT}^{xx} \rangle$  are parity conserving, and the other components such as  $\langle \lambda \rangle$ ,  $\langle S_T^x \rangle$ ,  $\langle S_{LT}^y \rangle$  and  $\langle S_{TT}^{xy} \rangle$  are parity violating. This implies that if we consider parity conserving reactions, only the  $F$ -terms survive and the  $\tilde{F}_i$ 's have to vanish. In this case we see that we have only  $\langle S_{LL} \rangle$ ,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^x \rangle$  and  $\langle S_{TT}^{xx} \rangle$  are non-zero. Other components such as  $\langle \lambda \rangle$ ,  $\langle S_T^x \rangle$ ,  $\langle S_{LT}^y \rangle$  and  $\langle S_{TT}^{xy} \rangle$  have to vanish.

In the case where transverse components are concerned, it is often useful to study different components with respect to the two transverse directions  $\vec{e}_n$  and  $\vec{e}_t$  defined as  $\vec{e}_n = \vec{p}_1 \times \vec{p}_2 / |\vec{p}_1 \times \vec{p}_2| = (-\sin \varphi, \cos \varphi)$  and  $\vec{e}_t = \vec{p}_{2T} / |\vec{p}_{2T}| = (\cos \varphi, \sin \varphi)$ , i.e. the normal and tangent of the hadron-hadron plane respectively. The corresponding components of the polarization are given by exactly the same equations such as Eqs. (3.73)-(3.74) with  $i = n$  or  $t$ . It can easily be shown that such components can also be obtained from Eqs. (3.73) and (3.74) with  $\varphi_\sigma$  being replaced by  $\varphi_\sigma - \varphi$  in the integrations given in Eqs. (3.75-3.78), e.g.,

$$\mathcal{F}_T^n = \int \frac{d\varphi_S}{\pi} \sin(\varphi_S - \varphi) \mathcal{F}_T, \quad (3.100)$$

$$\mathcal{F}_T^t = \int \frac{d\varphi_S}{\pi} \cos(\varphi_S - \varphi) \mathcal{F}_T, \quad (3.101)$$

$$\mathcal{F}_{LT}^n = \int \frac{d\varphi_{LT}}{\pi} \sin(\varphi_{LT} - \varphi) \mathcal{F}_{LT}, \quad (3.102)$$

$$\mathcal{F}_{LT}^t = \int \frac{d\varphi_{LT}}{\pi} \cos(\varphi_{LT} - \varphi) \mathcal{F}_{LT}, \quad (3.103)$$

$$\mathcal{F}_{TT}^{nn} = - \int \frac{d\varphi_{TT}}{\pi} \cos(2\varphi_{TT} - 2\varphi) \mathcal{F}_{TT}, \quad (3.104)$$

$$\mathcal{F}_{TT}^{nt} = \int \frac{d\varphi_{TT}}{\pi} \sin(2\varphi_{TT} - 2\varphi) \mathcal{F}_{TT}. \quad (3.105)$$

It will be also interesting to see the results after integrating over  $\varphi$ , we just pick the corresponding  $\cos(\varphi_\sigma - \varphi)$ - or  $\sin(\varphi_\sigma - \varphi)$ -terms. More precisely, we have,

$$\langle S_T^n \rangle = \frac{2}{3F_{Ut}} \left[ (1 + \cos^2 \theta) F_{1T}^{\sin(\varphi_S - \varphi)} + \sin^2 \theta F_{2T}^{\sin(\varphi_S - \varphi)} + \cos \theta F_{3T}^{\sin(\varphi_S - \varphi)} \right], \quad (3.106)$$

$$\langle S_T^t \rangle = \frac{2}{3F_{Ut}} \left[ (1 + \cos^2 \theta) \tilde{F}_{1T}^{\cos(\varphi_S - \varphi)} + \sin^2 \theta \tilde{F}_{2T}^{\cos(\varphi_S - \varphi)} + \cos \theta \tilde{F}_{3T}^{\cos(\varphi_S - \varphi)} \right], \quad (3.107)$$

$$\langle S_{LT}^n \rangle = \frac{2}{3F_{Ut}} \left[ (1 + \cos^2 \theta) \tilde{F}_{1LT}^{\sin(\varphi_{LT} - \varphi)} + \sin^2 \theta \tilde{F}_{2LT}^{\sin(\varphi_{LT} - \varphi)} + \cos \theta \tilde{F}_{3LT}^{\sin(\varphi_{LT} - \varphi)} \right], \quad (3.108)$$

$$\langle S_{LT}^t \rangle = \frac{2}{3F_{Ut}} \left[ (1 + \cos^2 \theta) F_{1LT}^{\cos(\varphi_{LT} - \varphi)} + \sin^2 \theta F_{2LT}^{\cos(\varphi_{LT} - \varphi)} + \cos \theta F_{3LT}^{\cos(\varphi_{LT} - \varphi)} \right], \quad (3.109)$$

$$\langle S_{TT}^{nm} \rangle = \frac{-2}{3F_{Ut}} \left[ (1 + \cos^2 \theta) F_{1TT}^{\cos(2\varphi_{TT} - 2\varphi)} + \sin^2 \theta F_{2TT}^{\cos(2\varphi_{TT} - 2\varphi)} + \cos \theta F_{3TT}^{\cos(2\varphi_{TT} - 2\varphi)} \right], \quad (3.110)$$

$$\langle S_{TT}^{nt} \rangle = \frac{2}{3F_{Ut}} \left[ (1 + \cos^2 \theta) \tilde{F}_{1TT}^{\sin(2\varphi_{TT} - 2\varphi)} + \sin^2 \theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT} - 2\varphi)} + \cos \theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT} - 2\varphi)} \right]. \quad (3.111)$$

It is interesting to see that all the average transverse polarizations w.r.t. the hadron-hadron plane take similar form in terms of the corresponding structure functions. We also see that in this case  $\langle S_T^n \rangle$ ,  $\langle S_{LT}^t \rangle$  and  $\langle S_{TT}^{nm} \rangle$  are parity conserving while  $\langle S_T^t \rangle$ ,  $\langle S_{LT}^n \rangle$  and  $\langle S_{TT}^{nt} \rangle$  are parity violating.

In experiments, it is usually very difficult to study azimuthal dependence and hadron polarization simultaneously. From the kinematic analysis given above, we see that we can either study the azimuthal asymmetries given by Eqs. (3.67-3.70) in the unpolarized case, or study the longitudinal hadron polarization in the helicity frame and transverse polarizations w.r.t. lepton-hadron plane or the hadron-hadron plane averaged over the azimuthal angle  $\varphi$  to study the corresponding structure functions as given by Eqs. (3.94-3.99) or Eqs. (3.106-3.111).

### C. Reduce to $e^+e^- \rightarrow VX$

It is also clear that if we consider the inclusive process  $e^+e^- \rightarrow VX$ , we should integrate over  $p_2$ , i.e. carrying out the integration  $\int d^3 p_2 / (2E_2)$ , to obtain the corresponding hadronic tensor and/or cross section. In this case, we obtain 3 for unpolarized, 3 for  $\lambda$ -, 3 for  $S_{LL}$ -, 4 for  $S_T$ -, 4 for  $S_{LT}$ - and 2 for  $S_{TT}$ -dependent part. The basic Lorentz tensors for the

hadronic tensor obtained in this case are given by,

$$h_{Ui,in}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_{1q}^\mu p_{1q}^\nu \right\}, \quad (3.112)$$

$$\tilde{h}_{U,in}^{A\mu\nu} = \varepsilon^{\mu\nu qp_1}, \quad (3.113)$$

$$h_{V,in}^{S\mu\nu} = \varepsilon^{\{\mu qp_1 S} p_{1q}^{\nu\}}, \quad (3.114)$$

$$\tilde{h}_{V,in}^{S\mu\nu} = \left\{ (q \cdot S) h_{Uj,in}^{S\mu\nu}, S_q^{\{\mu} p_{1q}^{\nu\}} \right\}, \quad (3.115)$$

$$h_{Vi,in}^{A\mu\nu} = \left\{ (q \cdot S) \tilde{h}_{U,in}^{A\mu\nu}, \varepsilon^{\{\mu qp_1 S} p_{1q}^{\nu\}} \right\}, \quad (3.116)$$

$$\tilde{h}_{V,in}^{A\mu\nu} = S_q^{\{\mu} p_{1q}^{\nu\}}, \quad (3.117)$$

$$h_{LLi,in}^{S\mu\nu} = S_{LL} h_{U,in}^{S\mu\nu}, \quad (3.118)$$

$$\tilde{h}_{LL,in}^{A\mu\nu} = S_{LL} \tilde{h}_{U,in}^{A\mu\nu}, \quad (3.119)$$

$$h_{LT,in}^{S\mu\nu} = S_{LT}^{\{\mu} p_{1q}^{\nu\}}, \quad (3.120)$$

$$\tilde{h}_{LT,in}^{S\mu\nu} = \varepsilon^{\{\mu qp_1 S_{LT} p_{1q}^{\nu\}}, \quad (3.121)$$

$$h_{LT,in}^{A\mu\nu} = S_{LT}^{\{\mu} p_{1q}^{\nu\}}, \quad (3.122)$$

$$\tilde{h}_{LT,in}^{A\mu\nu} = \varepsilon^{\{\mu qp_1 S_{LT} p_{1q}^{\nu\}}, \quad (3.123)$$

$$h_{TT,in}^{S\mu\nu} = S_{TT}^{\mu\nu}, \quad (3.124)$$

$$\tilde{h}_{TT,in}^{S\mu\nu} = \varepsilon^{\{\mu \alpha qp_1 S_{TT}^{\nu\}}. \quad (3.125)$$

There are totally 19 such independent basic Lorentz tensors, 10 of them are space reflection even and 9 of them are space reflection odd. We note in particular the spin-dependent time reversal odd term  $h_{V,in}^{S\mu\nu} = \varepsilon^{\{\mu qp_1 S} p_{1q}^{\nu\}}$  novel to deep-inelastic lepton-nucleon scattering (DIS) as discussed in [27]. This corresponds to single transverse polarization of  $V$  w.r.t. the lepton-hadron plane. There could be also parity violating transverse polarization in the lepton-hadron plane described by the last one in Eq. (3.115) i.e.  $\tilde{h}_{V3,in}^{S\mu\nu} = S_q^{\{\mu} p_{1q}^{\nu\}}$ .

The inclusive process  $e^+e^- \rightarrow VX$  can also be studied in the Helicity-GJ frame. Formerly, the differential cross section for  $e^+e^- \rightarrow VX$  takes exactly the same form as that for  $e^+e^- \rightarrow V\pi X$  integrated over  $\varphi$ . The corresponding inclusive structure functions just have one to one correspondence to those given by Eqs. (3.80-3.91). They are just equal to the counterparts in Eqs. (3.80-3.91) integrated over  $\xi_2$  and  $p_{2T}^2$ . In this case, we can study the longitudinal polarization and the transverse polarization with respect to the lepton-hadron plane that have similar expressions in terms of the structure functions as those given by Eqs. (3.92-3.99).

## IV. HADRONIC TENSOR IN TERMS OF FFS

We now calculate the hadronic tensor and differential cross section in the partonic picture at leading order in pQCD but with leading and twist-3 contributions. In this section we present the results obtained for the hadronic tensor.

In the partonic picture at the leading order in pQCD, we need to consider the contributions from the diagrams shown in Figs. 2 and 3 just as in [7] where spin-1/2 hadrons are considered. We need to perform the collinear expansion and pick up the results up to the order  $1/Q$  in order to get the twist-3

contributions. Collinear expansion was first proposed for inclusive process [28, 29] and has now been applied to all processes where one hadron is explicitly involved [15, 17, 30]. Systematic derivations have been given for such processes (for a recent short summary see e.g. [20]). However, for processes with no less than two hadrons are involved, systematic derivation for collinear expansion is still lacking. Usually, one just picks up terms up to  $1/Q$  from these diagrams [6–13, 16]. We do it in the same way in the following of this paper.

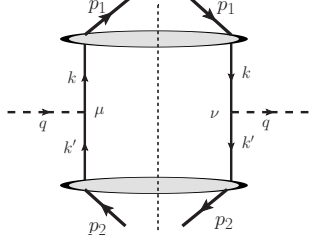


FIG. 2: Feynman diagram for  $Z \rightarrow V\pi X$  without gluon exchange that contributes at leading and higher twists.

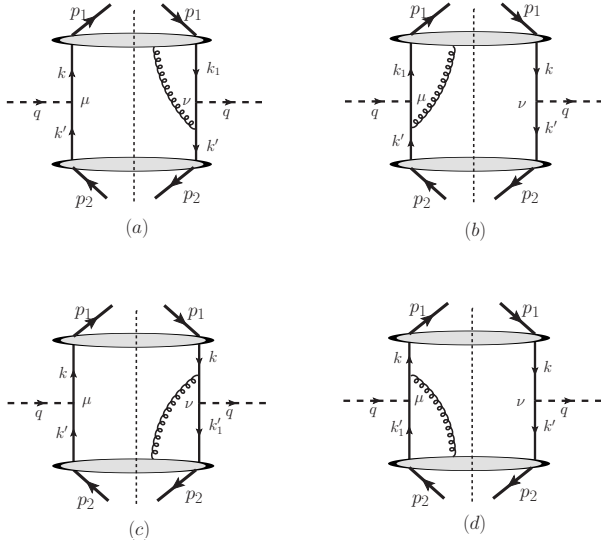


FIG. 3: Feynman diagrams for  $Z \rightarrow V\pi X$  with one gluon exchange that contribute at twist-3 and higher twists.

### A. Hadronic tensor in the collinear frame

The leading power contribution from Fig. 2 gives us the leading twist contribution where no transverse gluon exchange is involved. The longitudinal gluon exchanges lead to the gauge link that is needed to keep the quark-quark correlator gauge invariant. Up to twist-3, we need the next to the leading power contribution from Fig. 2 and also the leading power contributions from Fig. 3, where the quark-gluon-quark correlator is involved. We use the definition of the quark-gluon-quark correlator as given in Eq. (2.14) i.e. to use the

covariant derivative  $D$  instead of  $A$ . This is not only to use the simple relationships as given by Eqs. (2.16) and (2.17) but also to be consistent to the cases of  $e^+e^- \rightarrow V\bar{q}X$  and  $e^+e^- \rightarrow VX$  where collinear expansion has already been systematically proven [15, 17]. To do so, we need to pick up the corresponding  $k_\perp$ -terms from Fig. 2 and add them to those from Fig. 3. In this way, we obtain  $W_{\mu\nu} = \tilde{W}_{\mu\nu}^{(0)} + \tilde{W}_{\mu\nu}^{(1)} - \Delta\tilde{W}_{\mu\nu}^{(0)}$ . For the contribution  $\tilde{W}_{\mu\nu}^{(0)}$  from Fig. 2, we have,

$$\tilde{W}_{\mu\nu}^{(0)} = \frac{1}{p_1^+ p_2^-} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \text{Tr}[\Xi^{(0)}(z_1, k_\perp, p_1, S) \Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2) \Gamma_\nu]. \quad (4.1)$$

Corresponding to Fig. 3a, we have,

$$\tilde{W}_{\mu\nu}^{(1a)} = \frac{-1}{\sqrt{2} Q p_1^+ p_2^-} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \text{Tr}[\Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2) \gamma_\rho \not{q} \Gamma_\nu \Xi^{(1)\rho}(z_1, k_\perp, p_1, S)], \quad (4.2)$$

$$\Delta\tilde{W}_{\mu\nu}^{(0a)} = \frac{1}{\sqrt{2} Q p_1^+ p_2^-} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times k_\perp^\rho \text{Tr}[\Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2) \gamma_\rho \not{q} \Gamma_\nu \Xi^{(0)}(z_1, k_\perp, p_1, S)], \quad (4.3)$$

and similar for those from Figs. 3(b-d). The transverse momentum dependent quark-quark or quark-gluon-quark correlator,  $\Xi^{(0)}(z, k_\perp, p, S)$  or  $\Xi^{(1)\rho}(z, k_\perp, p, S)$  are given by Eq. (2.5) or (2.14) respectively. We use  $\bar{\Xi}$  to denote that for anti-quark fragmentation that differs from the corresponding one for quark by exchanging  $\psi$  and  $\bar{\psi}$  in the definition. Here as well as in the following of this paper, for explicitness, we consider only  $q \rightarrow VX$  and  $\bar{q} \rightarrow \pi X$ . The complete results should be the sum of these contributions and those from  $\bar{q} \rightarrow VX$  and  $q \rightarrow \pi X$ . The latter are just obtained simply by changing the  $\Xi$ 's to the corresponding  $\bar{\Xi}$ 's and  $\bar{\Xi}$ 's to the corresponding  $\Xi$ 's. Also a summation over the flavor of  $q$  is implicit.

We emphasize in particular that these expressions (4.1-4.3) are obtained from Figs. (2-3) and they are also straightforward extensions of the results obtained for  $e^+e^- \rightarrow V\bar{q}X$  which is a special case by setting  $|p_2, X\rangle$  as an anti-quark final state  $|k'\rangle$ . In the latter case  $\tilde{W}_{\mu\nu}^{(0)} - \Delta\tilde{W}_{\mu\nu}^{(0)}$  together reduces to the corresponding results of  $\tilde{W}_{\mu\nu}^{(0)}$  while  $\tilde{W}_{\mu\nu}^{(1)}$  reduces to the corresponding result directly.

To obtain the corresponding results for the hadronic tensors, we need to substitute the Lorentz decompositions of the quark-quark and quark-gluon-quark correlators as given by the equations in Appendix A into the above Eqs.(4.1-4.3) and carry out the traces. We note that all the decompositions of the quark-quark and quark-gluon-quark correlators are given in the collinear frame of the corresponding hadron, i.e. the direction of motion of the hadron is taken as the longitudinal direction. Hence, the most convenient frame to carry out the calculations of the hadronic tensor is the collinear frame of the hadron. Fortunately, in the case we discuss here, we have only two hadrons and we can make a Lorentz transformation into a frame where the two hadrons moving in the opposite directions. We call it the collinear frame of the two hadrons.



We first present the results of the hadronic tensor in this frame and then transform them into the Helicity-GJ-frame.

### 1. Hadronic tensor at twist-2

The leading twist contribution to the hadronic tensor comes solely from  $\tilde{W}_{\mu\nu}^{(0)}$  given by Eq. (4.1). To obtain the results, we insert the leading twist parts for the quark-quark correlator given in Appendix A. The unpolarized part and the vector polarization dependent parts are the same as those for spin-1/2 hadrons and can be found e.g. in [14]. We present here for completeness and for unification of notations. First of all, the simplest case, i.e. the unpolarized part is given by,

$$W_{\mu\nu}^{(0)U}(q, p_1, p_2) = \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ -(c_1^q g_{\perp\mu\nu} + i c_3^q \varepsilon_{\perp\mu\nu}) D_1(z_1, k_\perp) \bar{D}_1(z_2, k'_\perp) + \frac{4c_2^q}{M_1 M_2} (k_{\perp[\mu} k'_{\perp\nu]} - k_\perp \cdot k'_\perp g_{\perp\mu\nu}) H_1^\perp(z_1, k_\perp) \bar{H}_1^\perp(z_2, k'_\perp) \right\}, \quad (4.4)$$

where  $c_2^q = (c_V^q)^2 - (c_A^q)^2$ ;  $z_1 \approx \xi_1$  and  $z_2 \approx \xi_2$  up to  $1/Q$ . To make the results look more concise and explicit, we introduce the basic Lorentz tensors similar to those defined in [17], i.e.,

$$c_{\perp\mu\nu} = c_1^q g_{\perp\mu\nu} + i c_3^q \varepsilon_{\perp\mu\nu}, \quad (4.5)$$

$$\tilde{c}_{\perp\mu\nu} = c_3^q g_{\perp\mu\nu} + i c_1^q \varepsilon_{\perp\mu\nu}, \quad (4.6)$$

$$\alpha_{\perp\mu\nu}(a, b) = a_{\perp[\mu} b_{\perp\nu]} - (a_\perp \cdot b_\perp) g_{\perp\mu\nu}, \quad (4.7)$$

for two Lorentz vectors  $a$  and  $b$ . We will also omit the arguments of FFs in the expressions in the following of this paper. Since we are considering only the case of  $q \rightarrow VX$  and  $\bar{q} \rightarrow \pi X$ , this omission will not cause any ambiguity. The FFs defined via  $\Xi$ 's, i.e.  $D$ 's,  $G$ 's,  $E$ 's and  $H$ 's, are for  $q \rightarrow VX$  and have the arguments  $(z_1, k_\perp)$ , while those defined via  $\bar{\Xi}$ 's, i.e.  $\bar{D}$ 's,  $\bar{G}$ 's,  $\bar{E}$ 's and  $\bar{H}$ 's, are for  $\bar{q} \rightarrow \pi X$  and have the arguments  $(z_2, k'_\perp)$ . With such simplified notations, we have,

$$W_{\mu\nu}^{(0)U} = \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ -c_{\perp\mu\nu} D_1 \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(k, k') H_1^\perp \bar{H}_1^\perp \right\}. \quad (4.8)$$

We see that for the unpolarized part at twist-2, we have chiral even contribution from  $D_1$  convoluted with  $\bar{D}_1$  and chiral odd contribution from  $H_1^\perp$  convoluted with  $\bar{H}_1^\perp$ . We also note that for the chiral even contribution, there is a symmetric and an anti-symmetric part. However for the chiral odd contribution, there is only a symmetric part.

For the vector polarization dependent part, we write the longitudinally and transversely polarized parts separately. For the longitudinally polarized part, we have

$$W_{\mu\nu}^{(0)L} = \frac{4\lambda}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ \tilde{c}_{\perp\mu\nu} G_{1L} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(\tilde{k}', k) H_{1L}^\perp \bar{H}_1^\perp \right\}. \quad (4.9)$$

We see that besides the helicity  $\lambda$  factor, this takes quite similar form as that for the unpolarized part. Here, we have contributions from  $G_{1L}$  convoluted with  $\bar{D}_1$  and from  $H_{1L}^\perp$  with  $\bar{H}_1^\perp$ . For the transverse polarization dependent part, we have,

$$W_{\mu\nu}^{(0)T} = \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ \frac{k_\perp \cdot S_T}{M_1} [\tilde{c}_{\perp\mu\nu} G_{1T}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(\tilde{k}', k) H_{1T}^\perp \bar{H}_1^\perp] - \frac{\tilde{k}_\perp \cdot S_\perp}{M_1} c_{\perp\mu\nu} D_{1T}^\perp \bar{D}_1 + \frac{4c_2^q}{M_2} \alpha_{\perp\mu\nu}(\tilde{k}', S) H_{1T}^\perp \bar{H}_1^\perp \right\}. \quad (4.10)$$

Because there are two transverse directions, this part looks more complicated. We see clearly that we have both contributions in  $k_\perp$  or transverse to  $k_\perp$  (i.e. in  $\tilde{k}_\perp$ ) directions.

The  $S_{LL}$ -dependent part looks very much the same as the unpolarized part, i.e.,

$$W_{\mu\nu}^{(0)LL} = \frac{4S_{LL}}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ -c_{\perp\mu\nu} D_{1LL} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(k, k') H_{1LL}^\perp \bar{H}_1^\perp \right\}, \quad (4.11)$$

where we have the chiral even contribution from  $D_{1LL}$  convoluted with  $\bar{D}_1$  and chiral odd part from  $H_{1LL}^\perp$  with  $\bar{H}_1^\perp$ .

For the  $S_{LT}$ - and  $S_{TT}$ -dependent part, we have,

$$W_{\mu\nu}^{(0)LT} = \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ \frac{k_\perp \cdot S_{LT}}{M_1} [-c_{\perp\mu\nu} D_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(k, k') H_{1LT}^\perp \bar{H}_1^\perp] + \frac{\tilde{k}_\perp \cdot S_{LT}}{M_1} \tilde{c}_{\perp\mu\nu} G_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_2} \alpha_{\perp\mu\nu}(k', S_{LT}) H_{1LT}^\perp \bar{H}_1^\perp \right\}, \quad (4.12)$$

$$W_{\mu\nu}^{(0)TT} = \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \left\{ \frac{S_{TT}^{kk}}{M_1^2} [-c_{\perp\mu\nu} D_{1TT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(k, k') H_{1TT}^\perp \bar{H}_1^\perp] + \frac{S_{TT}^{\tilde{k}\tilde{k}}}{M_1^2} \tilde{c}_{\perp\mu\nu} G_{1TT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} \alpha_{\perp\mu\nu}(k', S_{TT}^k) H_{1TT}^\perp \bar{H}_1^\perp \right\}. \quad (4.13)$$

We see clearly the similarities and differences between them and the transverse polarization dependent part. We note once more that the chiral even contributions contain a symmetric and an anti-symmetric part given by the basic tensor  $c_{\perp\mu\nu}$  or  $\tilde{c}_{\perp\mu\nu}$  while the chiral odd contributions are always characterized by  $c_2^q$  and have only symmetric tensor  $\alpha_{\perp\mu\nu}$ .

### 2. Hadronic tensor at twist-3

The twist-3 contribution to the hadronic tensor comes from both Eq. (4.1) and (4.2). In Eq. (4.1), we either expand  $\Xi^{(0)}$  to leading twist and  $\bar{\Xi}^{(0)}$  to twist-3 or  $\bar{\Xi}^{(0)}$  to leading twist and

$\Xi^{(0)}$  to twist-3. In Eq. (4.2), we expand all the  $\Xi$ 's to their leading twist contribution. The equations are a bit longer than those at leading twist, we present as examples the results for the unpolarized and  $S_{LL}$ -dependent parts here but other parts in the appendix,

$$\begin{aligned}
W_{\mu\nu}^{(1)U} = & \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \\
& \times \left\{ \frac{1}{p_1^+} [\omega_{\mu\nu}(k) D^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}) G^\perp] \bar{D}_1 \right. \\
& - \frac{1}{p_2^-} D_1 [\omega_{\mu\nu}(k') \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') \bar{G}^\perp] \\
& - \frac{2c_2^q M_2}{M_1 p_2^-} H_1^\perp [2(k_n - k_{\bar{n}})_{[\mu\nu]} \bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]} \bar{E}] \\
& + \frac{2c_2^q M_1}{M_2 p_1^+} [2(k'_n - k'_{\bar{n}})_{[\mu\nu]} H + i(k'_n - k'_{\bar{n}})_{[\mu\nu]} E] \bar{H}_1^\perp \\
& \left. + \frac{\sqrt{2}}{Q} [\omega_{\mu\nu}(k', k) D_1 \bar{D}_1 - \frac{4c_2^q}{M_1 M_2} \omega_{\mu\nu}^{(n)}(k, k') H_1^\perp \bar{H}_1^\perp] \right\}, \quad (4.14)
\end{aligned}$$

where we introduce the short handed notations defined as,

$$a_{n[\mu\nu]} \equiv a_{\perp[\mu} n_{\nu]}, \quad a_{n[\mu\nu]} \equiv a_{\perp[\mu} n_{\nu]}, \quad (4.15)$$

$$\omega_{\mu\nu}(a, b) = c_1^q (a_n + b_{\bar{n}})_{[\mu\nu]} - i c_3^q (\tilde{a}_n + \tilde{b}_{\bar{n}})_{[\mu\nu]}, \quad (4.16)$$

$$\tilde{\omega}_{\mu\nu}(a, b) = c_3^q (a_n + b_{\bar{n}})_{[\mu\nu]} - i c_1^q (\tilde{a}_n + \tilde{b}_{\bar{n}})_{[\mu\nu]}, \quad (4.17)$$

$$\omega_{\mu\nu}^{(n)}(a, b) = (a_\perp^\perp b_{\bar{n}} + b_\perp^\perp a_n)_{[\mu\nu]}, \quad (4.18)$$

and  $\omega_{\mu\nu}(a) \equiv \omega_{\mu\nu}(a, -a)$ ,  $\tilde{\omega}_{\mu\nu}(a) \equiv \tilde{\omega}_{\mu\nu}(a, -a)$ .

The  $S_{LL}$ -dependent part looks very much similar, i.e.,

$$\begin{aligned}
W_{\mu\nu}^{(1)LL} = & \frac{4S_{LL}}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \\
& \times \left\{ \frac{1}{p_1^+} [\omega_{\mu\nu}(k) D_{LL}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}) G_{LL}^\perp] \bar{D}_1 \right. \\
& - \frac{1}{p_2^-} D_{1LL} [\omega_{\mu\nu}(k') \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') \bar{G}^\perp] \\
& - \frac{2c_2^q M_2}{M_1 p_2^-} H_{1LL}^\perp [2(k_n - k_{\bar{n}})_{[\mu\nu]} \bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]} \bar{E}] \\
& + \frac{2c_2^q M_1}{M_2 p_1^+} [2(k'_n - k'_{\bar{n}})_{[\mu\nu]} H_{LL} + i(k'_n - k'_{\bar{n}})_{[\mu\nu]} E_{LL}] \bar{H}_1^\perp \\
& \left. + \frac{\sqrt{2}}{Q} [\omega_{\mu\nu}(k', k) D_{1LL} \bar{D}_1 - \frac{4c_2^q}{M_1 M_2} \omega_{\mu\nu}^{(n)}(k, k') H_{1LL}^\perp \bar{H}_1^\perp] \right\}. \quad (4.19)
\end{aligned}$$

## B. Transform into the Helicity-GJ-frame

We now transform the hadronic tensor into Helicity-GJ-frame as described in Sec. IIIB 2. Since our goal is to express the hadronic tensor by FFs that are usually defined in the collinear way, we should just keep the FFs defined this way and transform the coefficients into the Helicity-GJ-frame

of the vector meson  $V$ . This is achieved by replacing the vectors and tensors in the hadronic tensor by their expressions in the Helicity-GJ-frame. Up to  $1/Q$ , we have [7, 13],

$$(k_{\perp\mu})_{coll} = k_{\perp\mu} - \sqrt{2} q_\perp \cdot k_\perp \bar{n}_\mu / Q + \dots, \quad (4.20)$$

$$(g_{\perp\mu\nu})_{coll} = g_{\perp\mu\nu} - \sqrt{2} q_{\bar{n}[\mu\nu]} / Q + \dots, \quad (4.21)$$

$$(\varepsilon_{\perp\mu\nu})_{coll} = \varepsilon_{\perp\mu\nu} + \sqrt{2} \tilde{q}_{\bar{n}[\mu\nu]} / Q + \dots, \quad (4.22)$$

and  $q_\perp = -p_{2T}/z_2 + \dots$ , where  $\dots$  are higher power suppressed terms. We see that the differences are all higher twist. It implies that the leading twist part is unchanged but there are additional twist-3 terms generated by transforming the twist-2 parts. E.g., for the unpolarized part, we have,

$$\begin{aligned}
\delta W_{\mu\nu}^{(1)U} = & \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \\
& \times \left\{ -(c_1^q q_{\bar{n}[\mu\nu]} - i c_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_1 \bar{D}_1 \right. \\
& \left. + \frac{4c_2^q}{M_1 M_2} (k_{\perp}^2 k'_{\bar{n}} + k_{\perp}^2 k_{\bar{n}})_{[\mu\nu]} H_1^\perp \bar{H}_1^\perp \right\}. \quad (4.23)
\end{aligned}$$

Others are given in the appendix B.

## V. STRUCTURE FUNCTIONS IN TERMS OF FFS

Making Lorentz contraction with the leptonic tensor, we obtain the cross section and the structure functions. The parton model results for the structure functions are given as convolution of the gauge invariant TMD FFs in the form,

$$\begin{aligned}
C[wD\bar{D}] = & \frac{1}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \\
& \times w(k_\perp, k'_\perp) D(z_1, k_\perp) \bar{D}(z_2, k'_\perp). \quad (5.1)
\end{aligned}$$

The weight  $w$  is a scalar function of  $k_\perp$  and  $k'_\perp$ . As in [14], we introduce the following dimensionless scalars,

$$w_0 = -k_\perp^2 / M_1^2, \quad (5.2)$$

$$\bar{w}_0 = -k'^2_\perp / M_2^2, \quad (5.3)$$

$$w_1 = -p_{2T} \cdot k_\perp / M_1 |\vec{p}_{2T}|, \quad (5.4)$$

$$\bar{w}_1 = -p_{2T} \cdot k'_\perp / M_2 |\vec{p}_{2T}|, \quad (5.5)$$

$$w_2 = -k_\perp \cdot k'_\perp / M_1 M_2. \quad (5.6)$$

Others are just functions of them and are given when needed.

### A. Structure functions at twist-2

We note that the twist-2 results presented here are for leading order in pQCD. Formally they just correspond to the results obtained from the naive or intuitive parton model.

We introduce a second digital in the subscript to specify the contributions at twist level, e.g.  $F_{jTi}^{\sin(\varphi_S - \varphi)}$ , and  $i = 1, 2, 3, \dots$  to specify the twist- $(i + 1)$  contributions. The unpolarized and vector polarization dependent parts can be derived from those

given in e.g. [14]. We list them here for completeness and comparison. We list only those non-zero structure functions. Those are not listed are zero at twist-2.

For the unpolarized part, we have,

$$F_{1U1} = 2c_1^e c_1^q C[D_1 \bar{D}_1], \quad (5.7)$$

$$F_{3U1} = 4c_3^e c_3^q C[D_1 \bar{D}_1], \quad (5.8)$$

$$F_{U1}^{\cos 2\varphi} = -8c_1^e c_2^q C[w_{hh} H_1^\perp \bar{H}_1^\perp], \quad (5.9)$$

where  $w_{hh} = 2w_1 \bar{w}_1 - w_2$ . The other 6  $F_U$ 's are zero at twist-2. We note in particular that there is a twist-2 contribution to  $\cos 2\varphi$  due to Collins function [4] but no such contribution to  $\cos \varphi$  or  $\sin \varphi$ .

The longitudinal polarization dependent part is very much the same as the unpolarized part. There are 3 non-zero  $F_L$ 's at twist-2, they are given by,

$$\tilde{F}_{1L1} = -2c_1^e c_3^q C[G_{1L} \bar{D}_1], \quad (5.10)$$

$$\tilde{F}_{3L1} = -4c_3^e c_1^q C[G_{1L} \bar{D}_1], \quad (5.11)$$

$$F_{L1}^{\sin 2\varphi} = -8c_1^e c_2^q C[w_{hh} H_{1L}^\perp \bar{H}_1^\perp]. \quad (5.12)$$

We see a one to one correspondence to the unpolarized terms. More precisely we have  $\tilde{F}_{jL1}$  just corresponds to  $F_{jU1}$  upon exchange of  $D_1$  to  $G_{1L}$  and  $F_{jL1}^{\sin 2\varphi}$  just corresponds to  $F_{U1}^{\cos 2\varphi}$  upon exchange of  $H_1^\perp$  to  $H_{1L}^\perp$ .

For the transverse polarization dependent part, we have,

$$F_{1T1}^{\sin(\varphi_S - \varphi)} = 2c_1^e c_1^q C[w_1 D_{1T}^\perp \bar{D}_1], \quad (5.13)$$

$$F_{3T1}^{\sin(\varphi_S - \varphi)} = 4c_3^e c_3^q C[w_1 D_{1T}^\perp \bar{D}_1], \quad (5.14)$$

$$\tilde{F}_{1T1}^{\cos(\varphi_S - \varphi)} = 2c_1^e c_3^q C[w_1 G_{1T}^\perp \bar{D}_1], \quad (5.15)$$

$$\tilde{F}_{3T1}^{\cos(\varphi_S - \varphi)} = 4c_3^e c_1^q C[w_1 G_{1T}^\perp \bar{D}_1], \quad (5.16)$$

$$F_{T1}^{\sin(\varphi_S + \varphi)} = -8c_1^e c_2^q C[\bar{w}_1 \mathcal{H}_{1T}^\perp \bar{H}_1^\perp], \quad (5.17)$$

$$F_{T1}^{\sin(\varphi_S - 3\varphi)} = -8c_1^e c_2^q C[w_{hh}' H_{1T}^\perp \bar{H}_1^\perp], \quad (5.18)$$

where  $w_{hh}' = w_1 w_{hh} - w_0 \bar{w}_1 / 2$ ,  $\mathcal{H}_{1T}^\perp$  is defined by Eq. (A20). We see that there are 6 non-zero transverse polarization dependent structure functions ( $F_T$  or  $\tilde{F}_T$ ) at twist-2, 4 of them are parity conserving and the other 2 are parity violating.

## B. Discussion about the twist-2 results

As we mentioned earlier in this paper, the twist-2 results presented here just correspond to the results obtained from the intuitive parton model with FFs defined in the gauge invariant form. Just as for the structure functions in inclusive deep-inelastic lepton-nucleon scattering (DIS) obtained using the original intuitive parton model, at the LO in pQCD and twist-2, the results exhibit a number of simple regularities (symmetries) such as Callan-Gross relation. To see these regularities more clearly, we list the leading twist results in Table I.

Indeed, from these results, we see that although there are 81

We see that among the 36 spin-independent and vector polarization dependent structure functions, 12 of them have twist-2 contributions while the other 24 are zero at twist -2. Among these 12 non-zero  $F$ 's, 8 are parity conserving 4 are parity violating, 8 of them correspond to azimuthal asymmetries.

For the tensor polarization dependent part, the results are much similar. First the  $S_{LL}$ -dependent part looks very much the same as the unpolarized part. There are only three non-zero  $F_{LL}$ 's at twist-2, they are given by,

$$F_{1LL1} = 2c_1^e c_1^q C[D_{1LL} \bar{D}_1], \quad (5.19)$$

$$F_{3LL1} = 4c_3^e c_3^q C[D_{1LL} \bar{D}_1], \quad (5.20)$$

$$F_{LL1}^{\cos 2\varphi} = -8c_1^e c_2^q C[w_{hh} H_{1LL}^\perp \bar{H}_1^\perp]. \quad (5.21)$$

The  $S_{LT}$ -dependent part is very much similar to the  $S_T$ -part. The 6 non-zeros are given by,

$$F_{1LT1}^{\cos(\varphi_{LT} - \varphi)} = -2c_1^e c_1^q C[w_1 D_{1LT}^\perp \bar{D}_1], \quad (5.22)$$

$$F_{3LT1}^{\cos(\varphi_{LT} - \varphi)} = -4c_3^e c_3^q C[w_1 D_{1LT}^\perp \bar{D}_1], \quad (5.23)$$

$$\tilde{F}_{1LT1}^{\sin(\varphi_{LT} - \varphi)} = -2c_1^e c_3^q C[w_1 G_{1LT}^\perp \bar{D}_1], \quad (5.24)$$

$$\tilde{F}_{3LT1}^{\sin(\varphi_{LT} - \varphi)} = -4c_3^e c_1^q C[w_1 G_{1LT}^\perp \bar{D}_1], \quad (5.25)$$

$$F_{LT1}^{\cos(\varphi_{LT} + \varphi)} = -8c_1^e c_2^q C[\bar{w}_1 \mathcal{H}_{1LT}^\perp \bar{H}_1^\perp], \quad (5.26)$$

$$F_{LT1}^{\cos(\varphi_{LT} - 3\varphi)} = 8c_1^e c_2^q C[w_{hh}' H_{1LT}^\perp \bar{H}_1^\perp]. \quad (5.27)$$

The  $S_{TT}$ -dependent part is similar to the  $S_T$ -part but the weights are different.

$$F_{1TT1}^{\cos(2\varphi_{TT} - 2\varphi)} = 2c_1^e c_1^q C[w_{dd}'' D_{1TT}^\perp \bar{D}_1], \quad (5.28)$$

$$F_{3TT1}^{\cos(2\varphi_{TT} - 2\varphi)} = 4c_3^e c_3^q C[w_{dd}'' D_{1TT}^\perp \bar{D}_1], \quad (5.29)$$

$$\tilde{F}_{1TT1}^{\sin(2\varphi_{TT} - 2\varphi)} = 2c_1^e c_3^q C[w_{dd}'' G_{1TT}^\perp \bar{D}_1], \quad (5.30)$$

$$\tilde{F}_{3TT1}^{\sin(2\varphi_{TT} - 2\varphi)} = 4c_3^e c_1^q C[w_{dd}'' G_{1TT}^\perp \bar{D}_1], \quad (5.31)$$

$$F_{TT1}^{\cos(2\varphi_{TT} - 4\varphi)} = -4c_1^e c_2^q C[w_{hh}'' H_{1TT}^\perp \bar{H}_1^\perp], \quad (5.32)$$

$$F_{TT1}^{\cos 2\varphi_{TT}} = 8c_1^e c_2^q C[w_2 H_{1TT}^\perp \bar{H}_1^\perp], \quad (5.33)$$

where  $w_{dd}'' = 2w_1 - w_0$ ,  $w_{hh}'' = w_0 w_2 - 4w_0 w_1 \bar{w}_1 + 4w_1^2 w_2 + 8w_1^3 \bar{w}_1$ , and  $H_{1TT}^\perp \equiv H_{1TT}^\perp + [k_\perp^2 + 8(k_\perp \cdot p_{2T})^2 / p_{2T}^2] H_{1TT}^\perp / 2M_1^2$ .

independent structure functions, a large part of them vanish at twist-2. Totally 27 of them are non-zero, among them 19 are parity conserved and 8 are parity violated. Furthermore we see following regularities.

(1) Among the 27 non-zero structure functions, 5 with  $c_1^e c_1^q$ , 5 with  $c_3^e c_3^q$  and 9 with  $c_1^e c_2^q$  are parity even, 4 with  $c_1^e c_3^q$  and 4 with  $c_3^e c_1^q$  are parity odd. This can be understood easily since from Eq. (3.2) we see that  $c_1^e$  symbolizes the symmetric parity conserving part and  $c_3^e$  the anti-symmetric parity violating part of the tensor.

(2) The non-vanishing structure functions are associated with either  $1 + \cos^2 \theta$ , or  $\cos \theta$  or  $\sin^2 \theta$ .

TABLE I: The 81 structure functions and their twist-2 results. The capital letter U, L, LL and so on in the first column denote the polarization of  $V$ . The  $\theta$ -dependence is common for all lines so we list it only in the first line. For the U, L and LL polarized parts, the space reflection behaviors of first three columns are different from the other six. For the others, i.e. T, LT and TT polarized parts, the whole line has the same reflection behavior and we use PC and PV to denote party conserved and parity violated respectively.

$\theta$ -dep.	$\sin \theta$	$\sin 2\theta$	$\sin^2 \theta$	$1 + \cos^2 \theta$	$\sin^2 \theta$	$\cos \theta$	$\sin \theta$	$\sin 2\theta$	$\sin^2 \theta$
$\varphi$ -dep.	$\sin \varphi$	$\sin 2\varphi$	$\sin^2 \varphi$	$1$	$\cos \varphi$	$\cos 2\varphi$			
U	$\tilde{F}_{1U}^{\sin \varphi}$	$\tilde{F}_{2U}^{\sin \varphi}$	$\tilde{F}_U^{\sin 2\varphi}$	$F_{1U}/2c_1^e c_1^q$	$F_{2U}$	$F_{3U}/4c_3^e c_3^q$	$F_{1U}^{\cos \varphi}$	$F_{2U}^{\cos \varphi}$	$F_U^{\cos 2\varphi}/8c_1^e c_2^q$
twist-2	0	0	0	$C[D_1 \bar{D}_1]$	0	$C[D_1 \bar{D}_1]$	0	0	$-C[w_{hh} H_1^\perp \bar{H}_1^\perp]$
L	$\tilde{F}_{1L}^{\sin \varphi}$	$\tilde{F}_{2L}^{\sin \varphi}$	$\tilde{F}_L^{\sin 2\varphi}/8c_1^e c_2^q$	$\tilde{F}_{1L}/2c_1^e c_3^q$	$\tilde{F}_{2L}$	$\tilde{F}_{3L}/4c_3^e c_1^q$	$\tilde{F}_{1L}^{\cos \varphi}$	$\tilde{F}_{2L}^{\cos \varphi}$	$\tilde{F}_L^{\cos 2\varphi}$
twist-2	0	0	$-C[w_{hh} H_{1L}^\perp \bar{H}_1^\perp]$	$-C[G_{1L} \bar{D}_1]$	0	$-C[G_{1L} \bar{D}_1]$	0	0	0
LL	$\tilde{F}_{1LL}^{\sin \varphi}$	$\tilde{F}_{2LL}^{\sin \varphi}$	$\tilde{F}_{LL}^{\sin 2\varphi}$	$F_{1LL}/2c_1^e c_1^q$	$F_{2LL}$	$F_{3LL}/4c_3^e c_3^q$	$F_{1LL}^{\cos \varphi}$	$F_{2LL}^{\cos \varphi}$	$F_{LL}^{\cos 2\varphi}/8c_1^e c_2^q$
twist-2	0	0	0	$C[D_{1LL} \bar{D}_1]$	0	$C[D_{1LL} \bar{D}_1]$	0	0	$-C[w_{hh} H_{1LL}^\perp \bar{H}_1^\perp]$
T-PC	$\tilde{F}_{1T}^{\sin \varphi_S}$	$\tilde{F}_{2T}^{\sin \varphi_S}$	$\tilde{F}_T^{\sin(\varphi_S + \varphi)}/8c_1^e c_2^q$	$F_{1T}^{\sin(\varphi_S - \varphi)}/2c_1^e c_1^q$	$F_{2T}^{\sin(\varphi_S - \varphi)}$	$F_{3T}^{\sin(\varphi_S - \varphi)}/4c_3^e c_3^q$	$\tilde{F}_{1T}^{\sin(\varphi_S - 2\varphi)}$	$\tilde{F}_{2T}^{\sin(\varphi_S - 2\varphi)}$	$\tilde{F}_T^{\sin(\varphi_S - 3\varphi)}/8c_1^e c_2^q$
$\varphi$ -dep.	$\sin \varphi_S$	$\sin(\varphi_S + \varphi)$		$\sin(\varphi_S - \varphi)$			$\sin(\varphi_S - 2\varphi)$		$\sin(\varphi_S - 3\varphi)$
twist-2	0	0	$-C[\bar{w}_1 \mathcal{H}_{1T}^\perp \bar{H}_1^\perp]$	$C[w_1 D_{1T}^\perp \bar{D}_1]$	0	$C[w_1 D_{1T}^\perp \bar{D}_1]$	0	0	$-C[w_{hh}' H_{1T}^\perp \bar{H}_1^\perp]$
T-PV	$\tilde{F}_{1T}^{\cos \varphi_S}$	$\tilde{F}_{2T}^{\cos \varphi_S}$	$\tilde{F}_T^{\cos(\varphi_S + \varphi)}$	$\tilde{F}_{1T}^{\cos(\varphi_S - \varphi)}/2c_1^e c_3^q$	$\tilde{F}_{2T}^{\cos(\varphi_S - \varphi)}$	$\tilde{F}_{3T}^{\cos(\varphi_S - \varphi)}/4c_3^e c_1^q$	$\tilde{F}_{1T}^{\cos(\varphi_S - 2\varphi)}$	$\tilde{F}_{2T}^{\cos(\varphi_S - 2\varphi)}$	$\tilde{F}_T^{\cos(\varphi_S - 3\varphi)}$
$\varphi$ -dep.	$\cos \varphi_S$	$\cos(\varphi_S + \varphi)$		$\cos(\varphi_S - \varphi)$			$\cos(\varphi_S - 2\varphi)$		$\cos(\varphi_S - 3\varphi)$
twist-2	0	0	0	$C[w_1 G_{1T}^\perp \bar{D}_1]$	0	$C[w_1 G_{1T}^\perp \bar{D}_1]$	0	0	0
LT-PC	$\tilde{F}_{1LT}^{\cos \varphi_{LT}}$	$\tilde{F}_{2LT}^{\cos \varphi_{LT}}$	$\tilde{F}_{LT}^{\cos(\varphi_{LT} + \varphi)}/8c_1^e c_2^q$	$F_{1LT}^{\cos(\varphi_{LT} - \varphi)}/2c_1^e c_1^q$	$F_{2LT}^{\cos(\varphi_{LT} - \varphi)}$	$F_{3LT}^{\cos(\varphi_{LT} - \varphi)}/4c_3^e c_3^q$	$\tilde{F}_{1LT}^{\cos(\varphi_{LT} - 2\varphi)}$	$\tilde{F}_{2LT}^{\cos(\varphi_{LT} - 2\varphi)}$	$\tilde{F}_{LT}^{\cos(\varphi_{LT} - 3\varphi)}/8c_1^e c_2^q$
$\varphi$ -dep.	$\cos \varphi_{LT}$	$\cos(\varphi_{LT} + \varphi)$		$\cos(\varphi_{LT} - \varphi)$			$\cos(\varphi_{LT} - 2\varphi)$		$\cos(\varphi_{LT} - 3\varphi)$
twist-2	0	0	$-C[\bar{w}_1 \mathcal{H}_{1LT}^\perp \bar{H}_1^\perp]$	$-C[w_1 D_{1LT}^\perp \bar{D}_1]$	0	$-C[w_1 D_{1LT}^\perp \bar{D}_1]$	0	0	$C[w_{hh}' H_{1LT}^\perp \bar{H}_1^\perp]$
LT-PV	$\tilde{F}_{1LT}^{\sin \varphi_{LT}}$	$\tilde{F}_{2LT}^{\sin \varphi_{LT}}$	$\tilde{F}_{LT}^{\sin(\varphi_{LT} + \varphi)}$	$\tilde{F}_{1LT}^{\sin(\varphi_{LT} - \varphi)}/2c_1^e c_3^q$	$\tilde{F}_{2LT}^{\sin(\varphi_{LT} - \varphi)}$	$\tilde{F}_{3LT}^{\sin(\varphi_{LT} - \varphi)}/4c_3^e c_1^q$	$\tilde{F}_{1LT}^{\sin(\varphi_{LT} - 2\varphi)}$	$\tilde{F}_{2LT}^{\sin(\varphi_{LT} - 2\varphi)}$	$\tilde{F}_{LT}^{\sin(\varphi_{LT} - 3\varphi)}$
$\varphi$ -dep.	$\sin \varphi_{LT}$	$\sin(\varphi_{LT} + \varphi)$		$\sin(\varphi_{LT} - \varphi)$			$\sin(\varphi_{LT} - 2\varphi)$		$\sin(\varphi_{LT} - 3\varphi)$
twist-2	0	0	0	$-C[w_1 G_{1LT}^\perp \bar{D}_1]$	0	$-C[w_1 G_{1LT}^\perp \bar{D}_1]$	0	0	0
TT-PC	$\tilde{F}_{1TT}^{\cos(2\varphi_{TT} - \varphi)}$	$\tilde{F}_{2TT}^{\cos(2\varphi_{TT} - \varphi)}$	$\tilde{F}_{TT}^{\cos 2\varphi_{TT}}/8c_1^e c_2^q$	$F_{1TT}^{\cos(2\varphi_{TT} - \varphi)}/2c_1^e c_1^q$	$F_{2TT}^{\cos(2\varphi_{TT} - \varphi)}$	$F_{3TT}^{\cos(2\varphi_{TT} - \varphi)}/4c_3^e c_3^q$	$\tilde{F}_{1TT}^{\cos(2\varphi_{TT} - 3\varphi)}$	$\tilde{F}_{2TT}^{\cos(2\varphi_{TT} - 3\varphi)}$	$\tilde{F}_{TT}^{\cos(2\varphi_{TT} - 4\varphi)}/4c_1^e c_2^q$
$\varphi$ -dep.	$\cos(2\varphi_{TT} - \varphi)$	$\cos 2\varphi_{TT}$		$\cos(2\varphi_{TT} - 2\varphi)$			$\cos(2\varphi_{TT} - 3\varphi)$		$\cos(2\varphi_{TT} - 4\varphi)$
twist-2	0	0	$C[w_2' \mathcal{H}_{1TT}^\perp \bar{H}_1^\perp]$	$C[w_{dd}' D_{1TT}^\perp \bar{D}_1]$	0	$C[w_{dd}' D_{1TT}^\perp \bar{D}_1]$	0	0	$-C[w_{hh}'' H_{1TT}^\perp \bar{H}_1^\perp]$
TT-PV	$\tilde{F}_{1TT}^{\sin(2\varphi_{TT} - \varphi)}$	$\tilde{F}_{2TT}^{\sin(2\varphi_{TT} - \varphi)}$	$\tilde{F}_{TT}^{\sin 2\varphi_{TT}}$	$\tilde{F}_{1TT}^{\sin(2\varphi_{TT} - 2\varphi)}/2c_1^e c_3^q$	$\tilde{F}_{2TT}^{\sin(2\varphi_{TT} - 2\varphi)}$	$\tilde{F}_{3TT}^{\sin(2\varphi_{TT} - 2\varphi)}/4c_3^e c_1^q$	$\tilde{F}_{1TT}^{\sin(2\varphi_{TT} - 3\varphi)}$	$\tilde{F}_{2TT}^{\sin(2\varphi_{TT} - 3\varphi)}$	$\tilde{F}_{TT}^{\sin(2\varphi_{TT} - 4\varphi)}$
$\varphi$ -dep.	$\sin(2\varphi_{TT} - \varphi)$	$\sin 2\varphi_{TT}$		$\sin(2\varphi_{TT} - 2\varphi)$			$\sin(2\varphi_{TT} - 3\varphi)$		$\sin(2\varphi_{TT} - 4\varphi)$
twist-2	0	0	0	$C[w_{dd}'' G_{1TT}^\perp \bar{D}_1]$	0	$C[w_{dd}'' G_{1TT}^\perp \bar{D}_1]$	0	0	0

For those associated with  $1 + \cos^2 \theta$  or  $\cos \theta$ , 5 with coefficient  $c_1^e c_1^q$  and 5 with  $c_3^e c_3^q$ . They are all from  $C[D\bar{D}]$ , i.e. fragmentations of unpolarized quark and are parity conserving. There are also 4 with coefficient  $c_1^e c_3^q$  and 4 with  $c_3^e c_1^q$ . They are all from  $C[G\bar{D}]$ , i.e. fragmentations of longitudinally polarized quark and unpolarized anti-quark and are parity violating.

Those associated with  $\sin^2 \theta$  all have coefficient  $c_1^e c_2^q$  and are from  $C[H\bar{H}]$ , i.e. transversely polarized quark and anti-quark.

To understand such regularities, we recall the result for the basic weak process  $e^+ e^- \rightarrow Z \rightarrow q\bar{q}$ . We recall that the differ-

ential cross section is [31],

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{\alpha^2}{4s} \chi [c_1^e c_1^q (1 + \cos^2 \theta) + 2c_3^e c_3^q \cos \theta], \quad (5.34)$$

and the produced quark (anti-quark) is longitudinally polarized and the polarization is given by,

$$P_q(\theta) = -\frac{c_1^e c_3^q (1 + \cos^2 \theta) + 2c_3^e c_1^q \cos \theta}{c_1^e c_1^q (1 + \cos^2 \theta) + 2c_3^e c_3^q \cos \theta}. \quad (5.35)$$

Furthermore, although the quark (anti-quark) is not transversely polarized, their transverse spin components are cor-



related. We define,

$$c_{nn}^q = \frac{|M_{n++}|^2 + |M_{n--}|^2 - |M_{n+-}|^2 - |M_{n-+}|^2}{|M_{n++}|^2 + |M_{n--}|^2 + |M_{n+-}|^2 + |M_{n-+}|^2}, \quad (5.36)$$

where  $M$  is the scattering amplitude,  $+$  or  $-$  denotes that the quark or anti-quark is in  $s_n = 1/2$  or  $-1/2$  state. We obtain that, for  $\vec{n}$  is in the normal of the production plane,

$$c_{nn}^q(\theta) = \frac{c_1^e c_2^q \sin^2 \theta}{c_1^e c_1^q (1 + \cos^2 \theta) + 2c_3^e c_3^q \cos \theta}, \quad (5.37)$$

which is in fact also true for any transverse direction  $\vec{n}$  if we replace  $\sin^2 \theta$  in the numerator by  $\sin^2 \theta \cos 2\varphi_n$  where  $\varphi_n$  is the azimuthal angle between  $\vec{n}$  and the normal of the production plane. In terms of  $y = (1 + \cos \theta)/2$ , we have,

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{\alpha^2}{2s} \chi T_0^q(y), \quad (5.38)$$

$$P_q(y) = T_1^q(y)/T_0^q(y), \quad (5.39)$$

$$c_{nn}^q(y) = c_1^e c_2^q C(y)/2T_0^q(y), \quad (5.40)$$

where  $T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y)$  is the relative production weight for flavor  $q$ ,  $T_1^q(y) = -c_1^e c_3^q A(y) + c_3^e c_1^q B(y)$ ;  $A(y)$ ,  $B(y)$  and  $C(y)$  are given in Sec. III B 2 by Eqs. (3.50-3.52). We see clearly why we have the regularities for the structure functions mentioned at the beginning of this point.

(3) It is also clear that if we consider  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$ , i.e. the electromagnetic process, we have,  $T_0^{q(em)}(y) = e_q^2 A(y)$ ,  $P_q^{(em)} = 0$ . The quark transverse spin correlation  $c_{nn}^{q(em)}(y) = C(y)/2A(y)$  independent of flavor of the quark. In this case, we will not have  $C[G\bar{D}]$ -terms but  $C[D\bar{D}]$  and  $C[H\bar{H}]$ -terms.

(4) If we integrate over  $p_2$ , we obtain the results for the inclusive process  $e^+e^- \rightarrow Z \rightarrow VX$ . The non-vanishing structure functions are,

$$z_1 F_{1U1,in} = 2c_1^e c_1^q D_1(z_1), \quad (5.41)$$

$$z_1 F_{3U1,in} = 4c_3^e c_3^q D_1(z_1), \quad (5.42)$$

$$z_1 \tilde{F}_{1L1,in} = -2c_1^e c_3^q G_{1L}(z_1), \quad (5.43)$$

$$z_1 \tilde{F}_{3L1,in} = -4c_3^e c_1^q G_{1L}(z_1), \quad (5.44)$$

$$z_1 F_{1LL,in} = 2c_1^e c_1^q D_{1LL}(z_1), \quad (5.45)$$

$$z_1 F_{3LL,in} = 4c_3^e c_3^q D_{1LL}(z_1). \quad (5.46)$$

All the others vanish at twist-2. This is consistent with the results obtained in [15]. We emphasize in particular that Callan-Gross relation in DIS now is replaced by  $F_{2U1,in} = 0$ , and all the structure functions associated with the transverse spin components vanish at leading twist.

### C. Twist-3 contributions

Among the 54 structure functions that vanish at twist-2, 36 have twist-3 contributions as the leading power contributions. The results are a bit lengthy so we present them as an appendix (see appendix C). We see that all the 36 structure

functions associated with  $\sin \theta$  and  $\sin 2\theta$  have twist-3 contributions as leading power contributions. Besides others, we have  $F_{1U2}^{\cos \varphi}$ ,  $F_{2U2}^{\cos \varphi}$ ,  $\tilde{F}_{1U2}^{\sin \varphi}$  and  $\tilde{F}_{2U2}^{\sin \varphi}$  in the unpolarized part, also  $F_{1T2}^{\sin \varphi_S}$ ,  $F_{2T2}^{\sin \varphi_S}$ ,  $\tilde{F}_{1T2}^{\cos \varphi_S}$  and  $\tilde{F}_{2T2}^{\cos \varphi_S}$  in the vector polarization dependent part. This means that at the twist-3 level there should be parity conserved azimuthal asymmetry  $\langle \cos \varphi \rangle_U$  and parity violated asymmetry  $\langle \sin \varphi \rangle_U$  in the unpolarized case and parity conserved transverse polarization in the normal direction of the lepton-hadron plane and parity violated component in the plane. We will discuss this more in next section.

## VI. AZIMUTHAL ASYMMETRIES AND HADRON POLARIZATIONS

### A. Azimuthal asymmetries

At leading twist and for unpolarized  $V$  (i.e. polarization is not measured), there is only one azimuthal asymmetry as given by Eq. (3.68), i.e.,

$$\langle \cos 2\varphi \rangle_U^{(0)} = -\frac{C(y) \sum_q c_1^e c_2^q C[w_{hh} H_1^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}. \quad (6.1)$$

This is the only leading twist azimuthal asymmetry in the unpolarized case due to Collins effect [4] and transverse spin correlation  $c_{nn}^q$  given by Eq. (5.37) for  $q\bar{q}$  produced via  $e^+e^-$  annihilation. Here, as well as in the following of this paper, when writing the expressions for azimuthal asymmetries and/or polarizations in terms of FFs, to avoid confusion, we include the summation over  $q$  explicitly but still keep the  $q \leftrightarrow \bar{q}$  terms implicitly and omit the flavor indices for the FFs.

If we could consider the polarization and azimuthal asymmetry simultaneously, we would have,

$$\langle \cos 2\varphi \rangle_{LL}^{(0)} = -\frac{C(y) \sum_q c_1^e c_2^q C[w_{hh} (H_1^\perp + S_{LL} H_{1LL}^\perp) \bar{H}_1^\perp]}{\sum_q T_0^q(y) C[(D_1 + S_{LL} D_{1LL}) \bar{D}_1]}, \quad (6.2)$$

$$\langle \sin 2\varphi \rangle_L^{(0)} = -\frac{\lambda C(y) \sum_q c_1^e c_2^q C[w_{hh} H_{1L}^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y) C(D_1 - \lambda G_{1L}) \bar{D}_1}. \quad (6.3)$$

Although it is academic since it will be very difficult to measure this asymmetry, it is interesting to see the existence of such asymmetry.

Up to twist-3, we have another two azimuthal asymmetries in the unpolarized case, i.e.,

$$\begin{aligned} \langle \cos \varphi \rangle_U^{(1)} &= -\frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}} \\ &\times \sum_q \left\{ T_2^q(y) (M_1 C[w_1 D^\perp z_2 \bar{D}_1] + M_2 C[\bar{w}_1 z_1 D_1 \bar{D}^\perp]) \right. \\ &\quad \left. + T_4^q(y) (M_1 C[\bar{w}_1 H z_2 \bar{H}_1^\perp] + M_2 C[w_1 z_1 H_1^\perp \bar{H}^\perp]) \right\}, \quad (6.4) \end{aligned}$$

$$\langle \sin \varphi \rangle_U^{(1)} = \frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}}$$

$$\times \sum_q \left\{ T_3^q(y) (M_1 C[w_1 G^\perp z_2 \bar{D}_1] - M_2 C[\bar{w}_1 z_1 D_1 \bar{G}^\perp]) + 2c_3^e c_2^q (M_1 C[\bar{w}_1 E z_2 \bar{H}_1^\perp] - M_2 C[w_1 z_1 H_1^\perp \bar{E}]) \right\}, \quad (6.5)$$

where  $D(y) = \sqrt{y(1-y)}$ ,  $T_2^q(y) = -c_3^e c_3^q + c_1^e c_1^q B(y)$ ,  $T_3^q(y) = c_3^e c_1^q - c_1^e c_3^q B(y)$ ,  $T_4^q(y) = 4c_1^e c_2^q B(y)$  and  $F_{U_t}^{(0)}$  is the twist-2 contribution to  $F_{U_t}$  and is given by,

$$F_{U_t}^{(0)} = 4 \sum_q T_0^q(y) C[D_1 \bar{D}_1]. \quad (6.6)$$

We see that they depend on several twist-3 FFs.

If we consider  $e^+ e^- \rightarrow \gamma^* \rightarrow V\pi X$ , we have,

$$\langle \cos 2\varphi \rangle_U^{(0,em)} = -\frac{C(y)}{A(y)} \frac{\sum_q e_q^2 C[w_{hh} H_1^\perp \bar{H}_1^\perp]}{\sum_q e_q^2 C[D_1 \bar{D}_1]}, \quad (6.7)$$

$$\begin{aligned} \langle \cos \varphi \rangle_U^{(1,em)} &= -\frac{2\tilde{B}(y)}{A(y)} \frac{1}{z_1 z_2 Q \sum_q e_q^2 C[D_1 \bar{D}_1]} \\ &\times \sum_q e_q^2 \left\{ M_1 C[w_1 D^\perp z_2 \bar{D}_1 + 4\bar{w}_1 H z_2 \bar{H}_1^\perp] \right. \\ &\quad \left. + M_2 C[\bar{w}_1 z_1 D_1 \bar{D}^\perp + 4w_1 z_1 H_1^\perp \bar{H}^\perp] \right\}, \quad (6.8) \end{aligned}$$

and  $\langle \sin \varphi \rangle_U^{(1,em)} = 0$ , where  $\tilde{B}(y) = \sqrt{y(1-y)}B(y)$ . In this case we have a non-zero azimuthal asymmetry  $\langle \cos 2\varphi \rangle_U^{(0,em)}$  at leading twist due to Collins effect [4] and a twist-3 asymmetry  $\langle \cos \varphi \rangle_U^{(0,em)}$  similar to Cahn effect [32] in deep-inelastic lepton-nucleon scattering.

## B. Hadron polarizations at twist-2

The polarization is in general dependent on  $\varphi$ . Experimentally it is much easier to consider the case where  $\varphi$  is integrated. In this case, at the leading twist, we have, for the longitudinal polarization,

$$\langle \lambda \rangle^{(0)} = \frac{2}{3} \frac{\sum_q P_q(y) T_0^q(y) C[G_{1L} \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}, \quad (6.9)$$

$$\langle S_{LL} \rangle^{(0)} = \frac{1}{2} \frac{\sum_q T_0^q(y) C[D_{1LL} \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}. \quad (6.10)$$

For transverse dependent components w.r.t. the hadron-hadron plane, we have,

$$\langle S_T^n \rangle^{(0)} = \frac{2}{3} \frac{\sum_q T_0^q(y) C[w_1 D_{1T}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}, \quad (6.11)$$

$$\langle S_T^t \rangle^{(0)} = -\frac{2}{3} \frac{\sum_q P_q(y) T_0^q(y) C[w_1 G_{1T}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}, \quad (6.12)$$

$$\langle S_{LT}^n \rangle^{(0)} = \frac{2}{3} \frac{\sum_q P_q(y) T_0^q(y) C[w_1 G_{1LT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}, \quad (6.13)$$

$$\langle S_{LT}^t \rangle^{(0)} = -\frac{2}{3} \frac{\sum_q T_0^q(y) C[w_1 D_{1LT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}, \quad (6.14)$$

$$\langle S_{TT}^{nn} \rangle^{(0)} = -\frac{2}{3} \frac{\sum_q T_0^q(y) C[w_{dd}^\perp D_{1TT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}, \quad (6.15)$$

$$\langle S_{TT}^{nt} \rangle^{(0)} = -\frac{2}{3} \frac{\sum_q P_q(y) T_0^q(y) C[w_{dd}^\perp G_{1TT}^\perp \bar{D}_1]}{\sum_q T_0^q(y) C[D_1 \bar{D}_1]}. \quad (6.16)$$

The transverse components w.r.t. the lepton-hadron plane are zero at the leading twist in  $\varphi$  integrated case.

If we consider  $e^+ e^- \rightarrow \gamma^* \rightarrow V\pi X$ , i.e. annihilate via electromagnetic interaction only, we have,

$$\langle S_{LL} \rangle^{(0,em)} = \frac{1}{2} \frac{\sum_q e_q^2 C[D_{1LL} \bar{D}_1]}{\sum_q e_q^2 C[D_1 \bar{D}_1]}, \quad (6.17)$$

$$\langle S_T^n \rangle^{(0,em)} = \frac{2}{3} \frac{\sum_q e_q^2 C[w_1 D_{1T}^\perp \bar{D}_1]}{\sum_q e_q^2 C[D_1 \bar{D}_1]}, \quad (6.18)$$

$$\langle S_{LT}^t \rangle^{(0,em)} = \frac{2}{3} \frac{\sum_q e_q^2 C[w_1 D_{1LT}^\perp \bar{D}_1]}{\sum_q e_q^2 C[D_1 \bar{D}_1]}, \quad (6.19)$$

$$\langle S_{TT}^{nn} \rangle^{(0,em)} = -\frac{2}{3} \frac{\sum_q e_q^2 C[w_{dd}^\perp D_{1TT}^\perp \bar{D}_1]}{\sum_q e_q^2 C[D_1 \bar{D}_1]}, \quad (6.20)$$

while the parity violating components,

$$\langle \lambda \rangle^{(0,em)} = \langle S_T^t \rangle^{(0,em)} = \langle S_{LT}^n \rangle^{(0,em)} = \langle S_{TT}^{nt} \rangle^{(0,em)} = 0. \quad (6.21)$$

We see in particular that the  $S_{LL}$ -component is non-zero at leading twist also in the parity conserved case. Parity conserving transverse components exist due to Sivers-type FFs such as  $D_{1T}^\perp$ ,  $D_{1LT}^\perp$  and  $D_{1TT}^\perp$  similar to the Sivers function  $f_{1T}^\perp$  in three dimensional PDFs [22].

For the inclusive process  $e^+ e^- \rightarrow Z \rightarrow VX$ , we have,

$$\langle \lambda \rangle_{in}^{(0)} = \sum_q 2P_q(y) T_0^q(y) G_{1L}(z_1) / \sum_q 3T_0^q(y) D_1(z_1), \quad (6.22)$$

$$\langle S_{LL} \rangle_{in}^{(0)} = \sum_q T_0^q(y) D_{1LL}(z_1) / \sum_q 2T_0^q(y) D_1(z_1), \quad (6.23)$$

while all the transverse components such as  $\langle S_T^i \rangle_{in}^{(0)}$ ,  $\langle S_{LT}^i \rangle_{in}^{(0)}$  and  $\langle S_{TT}^{ij} \rangle_{in}^{(0)}$  ( $i, j = x$  or  $y$ ) vanish at twist-2. We also see that  $\langle S_{LL} \rangle_{in}^{(0)}$  is non-zero also in parity conserved reactions while  $\langle \lambda \rangle_{in}^{(0)}$  exists only in parity violated case.

## C. Transverse polarizations with respect to the lepton-hadron plane at twist-3

As mentioned in Sec. VC, twist-3 contribution exists only for those structure functions that are zero at twist-2. They are the leading power contributions for the corresponding structure functions. In particular we see that there is no twist-3 contribution to the transverse components w.r.t. the hadron-hadron plane discussed in last subsection. However, for the transverse components w.r.t. the lepton-hadron plane, 4 of them, i.e.  $\langle S_T^x \rangle$ ,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^x \rangle$  and  $\langle S_{LT}^y \rangle$  have twist-3 contributions. They are determined by  $F_{jT2}^{\sin \varphi_S}$ ,  $\tilde{F}_{jT2}^{\cos \varphi_S}$ ,  $\tilde{F}_{jLT2}^{\sin \varphi_S}$  and  $F_{jLT2}^{\cos \varphi_S}$  given in appendix C respectively. The expressions can

easily be obtained by inserting these results into Eqs. (3.94-3.97) but are a bit lengthy so we omit them here. However we emphasize that if we consider  $e^+e^- \rightarrow \gamma^* \rightarrow V\pi X$ , the parity violating parts vanish and we have only the following two components,

$$\begin{aligned} \langle S_T^y \rangle^{(1,em)} &= \frac{8M_1 \tilde{B}(y)}{3z_1 z_2 Q A(y) \sum_q e_q^2 C[D_1 \bar{D}_1]} \\ &\times \sum_q e_q^2 \left\{ z_2 C[D_T^\perp \bar{D}_1 - 2 \frac{w_2}{M_1} H_T^{\perp-} \bar{H}_1^\perp] \right. \\ &\left. - \frac{z_1 M_2}{2M_1} C[w_2(D_{1T}^\perp \bar{D}^{\perp'} - G_{1T}^\perp \bar{G}^\perp) - 8\mathcal{H}_{1T}^\perp \bar{H}_1^{\perp'}] \right\}, \quad (6.24) \end{aligned}$$

$$\begin{aligned} \langle S_{LT}^x \rangle^{(1,em)} &= -\frac{8M_1 \tilde{B}(y)}{3z_1 z_2 Q A(y) \sum_q e_q^2 C[D_1 \bar{D}_1]} \\ &\times \sum_q e_q^2 \left\{ z_2 C[D_{LT}^\perp \bar{D}_1 - 2 \frac{w_2}{M_1} H_{LT}^{\perp+} \bar{H}_1^\perp] \right. \\ &\left. + \frac{z_1 M_2}{2M_1} C[w_2(D_{1LT}^\perp \bar{D}^{\perp'} + G_{1LT}^\perp \bar{G}^\perp) - 8\mathcal{H}_{1LT}^\perp \bar{H}_1^{\perp'}] \right\}. \quad (6.25) \end{aligned}$$

It is also interesting to see that these transverse components are defined w.r.t. the lepton-hadron plane and exist also in the inclusive process. For  $e^+e^- \rightarrow Z \rightarrow VX$ , we have,

$$\langle S_T^x \rangle_{in}^{(1)} = -\frac{8M_1 D(y)}{3z_1 Q} \frac{\sum_q T_3^q(y) G_T}{\sum_q T_0^q(y) D_1}, \quad (6.26)$$

$$\langle S_T^y \rangle_{in}^{(1)} = \frac{8M_1 D(y)}{3z_1 Q} \frac{\sum_q T_2^q(y) D_T}{\sum_q T_0^q(y) D_1}, \quad (6.27)$$

$$\langle S_{LT}^x \rangle_{in}^{(1)} = -\frac{8M_1 D(y)}{3z_1 Q} \frac{\sum_q T_2^q(y) D_{LT}}{\sum_q T_0^q(y) D_1}, \quad (6.28)$$

$$\langle S_{LT}^y \rangle_{in}^{(1)} = \frac{8M_1 D(y)}{3z_1 Q} \frac{\sum_q T_3^q(y) G_{LT}}{\sum_q T_0^q(y) D_1}. \quad (6.29)$$

We recall that  $\langle S_T^y \rangle$  is  $P$ -even and naive  $T$ -odd,  $\langle S_T^x \rangle$  is  $P$ -odd and naive  $T$ -even and  $\langle S_{LT}^y \rangle$  is  $P$ -odd and naive  $T$ -odd. Neither of these three can exist in deep-inelastic scattering such as  $e^-N \rightarrow e^-X$ . The only existing one is  $\langle S_{LT}^x \rangle$  which is both  $P$ - and  $T$ -even. We see also from table IV whether the corresponding FFs are  $T$ -odd or  $T$ -even which is consistent with the structure functions and/or the polarizations.

For  $e^+e^- \rightarrow \gamma^* \rightarrow VX$ , we have,

$$\langle S_T^y \rangle_{in}^{(1,em)} = \frac{8M_1 \tilde{B}(y)}{3z_1 Q A(y)} \frac{\sum_q e_q^2 D_T}{\sum_q e_q^2 D_1}, \quad (6.30)$$

$$\langle S_{LT}^x \rangle_{in}^{(1,em)} = -\frac{8M_1 \tilde{B}(y)}{3z_1 Q A(y)} \frac{\sum_q e_q^2 D_{LT}}{\sum_q e_q^2 D_1}, \quad (6.31)$$

and other two parity violating components are zero.

## VII. SUMMARY AND DISCUSSION

Three parts have been presented in this paper: A summary of results of a general decomposition of the quark-quark correlator that leads to the operator definition of TMD FFs, a

general kinematical analysis for  $e^+e^- \rightarrow V\pi X$  and a complete twist-3 calculation based on the partonic picture at leading order in pQCD. We summarize the main results in the following.

(1) We presented the results of general decomposition of quark-quark correlator for fragmentation of quark to spin-1 hadron. The correlator is expressed as a sum of a spin independent, a vector polarization dependent and a tensor polarization dependent part. Formally, the spin independent part is identical to that for spin-0 hadrons, the vector polarization dependent part is the same as that for spin-1/2 hadrons, while the tensor polarization dependent part is novel for spin-1 hadrons. The decomposition leads to totally 72 TMD FFs, 8 for spin independent, 24 for the vector polarization dependent and the other 40 for the tensor polarization dependent part. Among them, 18 contribute at leading twist, 36 at twist-3 and the other 18 at twist-4; half of them (36) are  $T$ -odd, the other half are  $T$ -even; also half are  $\chi$ -odd and the other half are  $\chi$ -even.

(2) These TMD FFs are used in describing the semi-inclusive high energy reaction (see e.g. [17]). We note that usually for a complete description of a semi-inclusive reaction, the quark-quark correlator is not sufficient. One usually needs quark- $j$ -gluon-quark correlator, too ( $j = 1, 2, \dots$  represents the number of gluons). They contribute at higher twist starting at twist- $(j+2)$ . For example, to make a complete calculation up to twist-3, besides the quark-quark correlator, one needs the quark-gluon-quark correlator. These contributions should be taken into account simultaneously. It is also important to note that because of the QCD equation of motion, they are often not independent and relationships obtained from QCD equation of motion should be used.

(3) We presented also the results for a general kinematic analysis for  $e^+e^- \rightarrow V\pi X$ . This process is in general described by 81 structure functions, 42 are parity conserving and 39 are parity violating. The azimuthal asymmetries and hadron polarizations are in general coupled with each other and are described by the corresponding structure functions. In practice, it is much simpler to study the azimuthal asymmetries in the unpolarized case and hadron polarizations averaged over the azimuthal angle  $\varphi$ . For unpolarized hadrons, there are 4 azimuthal asymmetries i.e.  $\langle \cos \varphi \rangle_U$ ,  $\langle \sin \varphi \rangle_U$ ,  $\langle \cos 2\varphi \rangle_U$ , and  $\langle \sin 2\varphi \rangle_U$ . The two cos-asymmetries are parity conserving while the two sin-asymmetries are parity violating.

(4) The hadron polarizations are most conveniently studied in the helicity Gottfried-Jackson frame. Here, we have two longitudinal components  $\langle \lambda \rangle$  and  $\langle S_{LL} \rangle$  defined in the helicity basis, and 6 transverse components that can be defined either w.r.t. the lepton-hadron plane, i.e.,  $\langle S_T^x \rangle$ ,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^x \rangle$ ,  $\langle S_{LT}^y \rangle$ ,  $\langle S_{TT}^{xx} \rangle$  and  $\langle S_{TT}^{xy} \rangle$ , or w.r.t the hadron-hadron plane i.e.  $\langle S_T^n \rangle$ ,  $\langle S_T^t \rangle$ ,  $\langle S_{LT}^n \rangle$ ,  $\langle S_{LT}^t \rangle$ ,  $\langle S_{TT}^{nn} \rangle$  and  $\langle S_{TT}^{nt} \rangle$ . In the case of averaging over  $\varphi$ , they correspond to different structure functions as given by Eqs. (3.94-3.99) and Eqs. (3.106-3.111) respectively. Half of them are parity conserving while the other half are parity violating.

(5) The results obtained in partonic picture at LO pQCD up to twist-3 are also presented in terms of the gauge invariant FFs. These results show that at leading twist there are 27 non-vanishing structure functions 19 correspond to parity conserving and 8 are parity violating. We have also 36 structure

functions that have twist-3 as leading power contributions.

(6) For unpolarized hadrons, there is only one azimuthal asymmetry  $\langle \cos 2\varphi \rangle$  at leading twist due to Collins effect [4] in fragmentation and transverse spin correlation  $c_{nn}^q$  given by Eq. (5.37) in  $e^+e^-$ -annihilations, and two twist-3 asymmetries  $\langle \cos \varphi \rangle$  and  $\langle \sin \varphi \rangle$ , the former is similar to the Cahn effect [32] in DIS and the latter exists only in parity violating reactions.

(7) Longitudinal components of hadron polarization  $\langle \lambda \rangle$  and  $\langle S_{LL} \rangle$  exist at leading twist as given by Eqs. (6.9-6.10). While the former depends on the initial polarization  $P_q$  of the quark produced at the  $e^+e^-$  annihilation vertex and exists only in weak interaction processes, the latter is independent of  $P_q$  and exists also in electromagnetic processes.

(8) Transverse components  $\langle S_T^n \rangle$ ,  $\langle S_T^t \rangle$ ,  $\langle S_{LT}^n \rangle$ ,  $\langle S_{LT}^t \rangle$ ,  $\langle S_{TT}^{nn} \rangle$  and  $\langle S_{TT}^{tt} \rangle$  w.r.t. the hadron-hadron plane exist at leading twist given by Eqs. (6.11-6.16). Among them  $\langle S_T^n \rangle$ ,  $\langle S_{LT}^n \rangle$  and  $\langle S_{TT}^{nn} \rangle$  are parity conserving and  $\langle S_T^t \rangle$ ,  $\langle S_{LT}^t \rangle$  and  $\langle S_{TT}^{tt} \rangle$  are parity violating.

(9) There are also twist-3 transverse components  $\langle S_T^x \rangle$ ,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^x \rangle$ ,  $\langle S_{LT}^y \rangle$ ,  $\langle S_{TT}^{xx} \rangle$  and  $\langle S_{TT}^{yy} \rangle$  w.r.t. lepton-hadron plane. They are determined by the corresponding twist-3 FFs as given by Eqs. (6.26-6.29). Similarly,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^y \rangle$  and  $\langle S_{TT}^{yy} \rangle$  are parity conserving and  $\langle S_T^x \rangle$ ,  $\langle S_{LT}^x \rangle$  and  $\langle S_{TT}^{xx} \rangle$  are parity violating.

(10) For inclusive reaction  $e^+e^- \rightarrow VX$ , we can only study  $z$ -dependence. Kinematically, the hadronic tensor and/or cross section take the same form as that of the inclusive reaction  $e^+e^- \rightarrow V\pi X$  averaged over  $\varphi$ . We have two longitudinal components of polarization i.e.  $\langle \lambda \rangle$  and  $\langle S_{LL} \rangle$  at leading twist. In particular we have 4 transverse components  $\langle S_T^x \rangle$ ,  $\langle S_T^y \rangle$ ,  $\langle S_{LT}^x \rangle$ ,  $\langle S_{LT}^y \rangle$  at twist-3. Three of them are either  $T$ -odd or  $P$ -odd and do not exist in deep-inelastic scattering such as  $e^-h \rightarrow e^-X$ . The only both  $P$  and  $T$ -even one is  $\langle S_{LT}^x \rangle$ .

Finally, we would like to emphasize in particular that, in experiments, different components of the (vector) polarizations of octet hyperons such as  $\Lambda$ ,  $\Sigma^\pm$  and  $\Xi^{0,-}$  and those of the tensor polarizations of vector mesons such as  $\rho$  and  $K^*$  can be measured in a conceptually simple way. Polarizations of these hyperons can be measured by studying the angular distributions of the decay products of their spin self analyzing parity violating decays. All the five independent components of the tensor polarization,  $S_{LL}$ ,  $S_{LT}^x$ ,  $S_{LT}^y$ ,  $S_{TT}^{xx}$  and  $S_{TT}^{yy}$ , of these vector mesons can also be measured via the angular distributions in their strong decays into two pseudoscalar mesons [21]. Such measurements have also been carried out in past in different high energy reactions. Transverse polar-

izations of different hyperons have been observed in unpolarized hadron-hadron, hadron-nucleus collisions [23], in  $e^+e^-$ -annihilations [33] and lepton-hadron reactions [34] that correspond to the Sivers type FF  $D_{1T}^\perp$  and higher twist addenda to it. We see in particular that in experiments with  $e^+e^-$  annihilation at high energies where FFs can be best studied, measurements have been carried out e.g. at LEP on longitudinal polarization of  $\Lambda$  hyperon production [35, 36] by ALEPH and OPAL collaborations, and also on the spin alignment  $\rho_{00} = (1 - 2S_{LL})/3$  for vector mesons such as  $K^*$ ,  $\rho$  and so on [37–39]. Results for  $z$  dependences have been obtained in both cases. Even non-diagonal components (corresponds to higher twist contributions only) have also been measured [37–39]. The data available are definitely still far from enough to limit the precise forms of the FFs involved. They have however provided important hints for the corresponding components and have attracted much attention theoretically. Many phenomenological model studies have been carried out in last few years [40–58].

Recent measurements have been carried out on azimuthal asymmetries for two hadron production by BELLE, BABAR and BES III Collaborations [59–63]. They provide useful constraints on Collins function [64, 65]. Presently, related measurements can be and are being carried out e.g. in  $pp$ -collisions by STAR at RHIC, and in the existing  $e^+e^-$  colliders such as BELLE at KEK and BES at BEPC [66]. They can certainly also be studied in future  $e^+e^-$  colliders at high energies, electron-ion colliders discussed in the community [67]. We would in particular like to note that usually the production rates of vector mesons are much higher than hyperons in high energy reactions. Hence, we expect that studies of vector meson tensor polarization might provide us a more sensitive window to study polarization effects in fragmentation process in particular and to develop QCD theory in general.

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### Appendix A: Fragmentation functions defined via quark-quark correlator

We make a full list of the TMD FFs defined via quark-quark correlator in this appendix.



## 1. The spin independent part

The general decomposition of the spin independent part of the quark-quark correlator is given by,

$$z\Xi^{U(0)}(z, k_{F\perp}; p) = ME(z, k_{F\perp}), \quad (\text{A1})$$

$$z\tilde{\Xi}^{U(0)}(z, k_{F\perp}; p) = 0, \quad (\text{A2})$$

$$z\Xi_\alpha^{U(0)}(z, k_{F\perp}; p) = p^+ \bar{n}_\alpha D_1(z, k_{F\perp}) + k_{F\perp\alpha} D^\perp(z, k_{F\perp}) + \frac{M^2}{p^+} n_\alpha D_3(z, k_{F\perp}), \quad (\text{A3})$$

$$z\tilde{\Xi}_\alpha^{U(0)}(z, k_{F\perp}; p) = -\tilde{k}_{F\perp\alpha} G^\perp(z, k_{F\perp}), \quad (\text{A4})$$

$$z\Xi_{\rho\alpha}^{U(0)}(z, k_{F\perp}; p) = -\frac{p^+}{M} \bar{n}_{[\rho} \tilde{k}_{F\perp\alpha]} H_1^\perp(z, k_{F\perp}) + M \varepsilon_{\perp\rho\alpha} H(z, k_{F\perp}) - \frac{M}{p^+} n_{[\rho} \tilde{k}_{F\perp\alpha]} H_3^\perp(z, k_{F\perp}). \quad (\text{A5})$$

Here, we note in particular that, compared with the corresponding  $\bar{n}$  component, the  $n_\perp$  and  $n$  components are suppressed by  $M/p^+$  and  $(M/p^+)^2$  and contribute at twist-3 and twist-4 respectively. If we integrate over  $d^2 k_{F\perp}$ , terms with  $k_{F\perp}$  odd Lorentz structures vanish and we obtain,

$$z\Xi^{U(0)}(z; p) = ME(z), \quad z\tilde{\Xi}^{U(0)}(z; p) = 0, \quad (\text{A6})$$

$$z\Xi_\alpha^{U(0)}(z; p) = p^+ \bar{n}_\alpha D_1(z) + \frac{M^2}{p^+} n_\alpha D_3(z), \quad z\tilde{\Xi}_\alpha^{U(0)}(z; p) = 0, \quad (\text{A7})$$

$$z\Xi_{\rho\alpha}^{U(0)}(z; p) = M \varepsilon_{\perp\rho\alpha} H(z), \quad (\text{A8})$$

where the one-dimensional FF is just equal to the corresponding three-dimensional one integrated over  $d^2 k_{F\perp}$  such as,

$$D_1(z) = \int \frac{d^2 k_{F\perp}}{(2\pi)^2} D_1(z, k_{F\perp}) = z \sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+ \xi^- / z} \langle p, S; X | \bar{\psi}(\xi^-) \mathcal{L}(\xi^-; \infty) | 0 \rangle \frac{\gamma^+}{4} \langle 0 | \mathcal{L}^+(0; \infty) \psi(0) | p, S; X \rangle. \quad (\text{A9})$$

The factor  $z$  before  $\Xi^{(0)}$  on the left-hand-side of Eqs. (A1-A5) is needed so that  $D_1(z)$  obtained this way is the number density for a quark fragmentation into a specified hadron. However, when polarization is involved, we note the difference: While for phenomenologically defined  $D_1(z)$ , a sum over spin of  $h$  and an average over the spin of quark is understood, for  $D_1(z)$  defined via quark-quark correlator as given by Eq. (A9), we have an average over hadron spin and a sum over quark spin. Hence  $D_1(z)$  is identical in the two cases only for spin 1/2 hadrons.

## 2. Vector polarization dependent part

We build the  $S$ -dependent basic Lorentz covariants with the corresponding properties under space reflection as demanded and obtain the general decomposition of the  $S$ -dependent part of the quark-quark correlator as,

$$z\Xi^{V(0)}(z, k_{F\perp}; p, S) = (\tilde{k}_{F\perp} \cdot S_T) E_T^\perp(z, k_{F\perp}), \quad (\text{A10})$$

$$z\tilde{\Xi}^{V(0)}(z, k_{F\perp}; p, S) = M \left[ \lambda E_L(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} E_T'^\perp(z, k_{F\perp}) \right], \quad (\text{A11})$$

$$\begin{aligned} z\Xi_\alpha^{V(0)}(z, k_{F\perp}; p, S) &= p^+ \bar{n}_\alpha \frac{\tilde{k}_{F\perp} \cdot S_T}{M} D_{1T}^\perp(z, k_{F\perp}) - M \tilde{S}_{T\alpha} D_T(z, k_{F\perp}) \\ &\quad - \tilde{k}_{F\perp\alpha} \left[ \lambda D_L^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} D_T^\perp(z, k_{F\perp}) \right] + \frac{M}{p^+} n_\alpha (\tilde{k}_{F\perp} \cdot S_T) D_{3T}^\perp(z, k_{F\perp}), \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} z\tilde{\Xi}_\alpha^{V(0)}(z, k_{F\perp}; p, S) &= p^+ \bar{n}_\alpha \left[ \lambda G_{1L}(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} G_{1T}^\perp(z, k_{F\perp}) \right] - M S_{T\alpha} G_T(z, k_{F\perp}) \\ &\quad - k_{F\perp\alpha} \left[ \lambda G_L^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} G_T^\perp(z, k_{F\perp}) \right] + \frac{M^2}{p^+} n_\alpha \left[ \lambda G_{3L}(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} G_{3T}^\perp(z, k_{F\perp}) \right], \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} z\Xi_{\rho\alpha}^{V(0)}(z, k_{F\perp}; p, S) &= p^+ \bar{n}_{[\rho} S_{T\alpha]} H_{1T}(z, k_{F\perp}) + \frac{p^+}{M} \bar{n}_{[\rho} k_{F\perp\alpha]} \left[ \lambda H_{1L}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} H_{1T}^\perp(z, k_{F\perp}) \right] \\ &\quad + k_{F\perp[\rho} S_{T\alpha]} H_T^\perp(z, k_{F\perp}) + M \bar{n}_{[\rho} n_{\alpha]} \left[ \lambda H_L(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} H_T^\perp(z, k_{F\perp}) \right] \\ &\quad + \frac{M^2}{p^+} n_{[\rho} S_{T\alpha]} H_{3T}(z, k_{F\perp}) + \frac{M}{p^+} n_{[\rho} k_{F\perp\alpha]} \left[ \lambda H_{3L}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} H_{3T}^\perp(z, k_{F\perp}) \right]. \end{aligned} \quad (\text{A14})$$

If we integrate over  $d^2k_{F\perp}$ , only 8 terms survive, i.e.,

$$z\Xi^{V(0)}(z; p, S) = 0, \quad (\text{A15})$$

$$z\tilde{\Xi}^{V(0)}(z; p, S) = \lambda M E_L(z), \quad (\text{A16})$$

$$z\Xi_\alpha^{V(0)}(z; p, S) = -M\tilde{S}_{T\alpha}D_T(z), \quad (\text{A17})$$

$$z\tilde{\Xi}_\alpha^{V(0)}(z; p, S) = \lambda p^+ \bar{n}_\alpha G_{1L}(z) - MS_{T\alpha}G_T(z) + \lambda \frac{M^2}{p^+} n_\alpha G_{3L}(z), \quad (\text{A18})$$

$$z\tilde{\Xi}_{\rho\alpha}^{V(0)}(z; p, S) = p^+ \bar{n}_{[\rho} S_{T\alpha]} H_{1T}(z) - \lambda M \bar{n}_{[\rho} n_{\alpha]} H_L(z) + \frac{M^2}{p^+} n_{[\rho} S_{T\alpha]} H_{3T}(z), \quad (\text{A19})$$

where the one-dimensional FF in the longitudinally polarized case is just equal to the corresponding three-dimensional FF integrated over  $d^2k_{F\perp}$ , while in the transversely polarized case, we have,

$$K_T(z) = \int \frac{d^2k_{F\perp}}{(2\pi)^2} \mathcal{K}_T^\perp(z, k_{F\perp}), \quad \mathcal{K}_T^\perp(z, k_{F\perp}) \equiv K_T(z, k_{F\perp}) + \frac{k_{F\perp}^2}{2M^2} K_T^\perp(z, k_{F\perp}), \quad (\text{A20})$$

for the transverse polarization dependent FFs such as  $K_T = D_T, G_T, H_{1T}$  or  $H_{3T}$ , and similar for the  $S_{LT}$ -dependent part in the following.

### 3. Tensor polarization dependent part

The most general decomposition for the tensor polarization dependent part is given by,

$$z\Xi^{T(0)}(z, k_{F\perp}; p, S) = M \left[ S_{LL} E_{LL}(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} E_{LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} E_{TT}^\perp(z, k_{F\perp}) \right], \quad (\text{A21})$$

$$z\tilde{\Xi}^{T(0)}(z, k_{F\perp}; p, S) = M \left[ \frac{\tilde{k}_{F\perp} \cdot S_{LT}}{M} E_{LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_F k_F}}{M^2} E_{TT}^\perp(z, k_{F\perp}) \right], \quad (\text{A22})$$

$$\begin{aligned} z\Xi_\alpha^{T(0)}(z, k_{F\perp}; p, S) &= p^+ \bar{n}_\alpha \left[ S_{LL} D_{1LL}(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} D_{1LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} D_{1TT}^\perp(z, k_{F\perp}) \right] \\ &\quad + MS_{LT\alpha} D_{LT}(z, k_{F\perp}) + S_{TT\alpha}^{k_F} D_{TT}^\perp(z, k_{F\perp}) \\ &\quad + k_{F\perp\alpha} \left[ S_{LL} D_{LL}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} D_{LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} D_{TT}^\perp(z, k_{F\perp}) \right] \\ &\quad + \frac{M^2}{p^+} n_\alpha \left[ S_{LL} D_{3LL}(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} D_{3LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} D_{3TT}^\perp(z, k_{F\perp}) \right], \end{aligned} \quad (\text{A23})$$

$$\begin{aligned} z\tilde{\Xi}_\alpha^{T(0)}(z, k_{F\perp}; p, S) &= p^+ \bar{n}_\alpha \left[ \frac{\tilde{k}_{F\perp} \cdot S_{LT}}{M} G_{1LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_F k_F}}{M^2} G_{1TT}^\perp(z, k_{F\perp}) \right] \\ &\quad - M\tilde{S}_{LT\alpha} G_{LT}(z, k_{F\perp}) - \tilde{S}_{TT\alpha}^{k_F} G_{TT}^\perp(z, k_{F\perp}) \\ &\quad - \tilde{k}_{F\perp\alpha} \left[ S_{LL} G_{LL}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} G_{LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} G_{TT}^\perp(z, k_{F\perp}) \right] \\ &\quad + \frac{M^2}{p^+} n_\alpha \left[ \frac{\tilde{k}_{F\perp} \cdot S_{LT}}{M} G_{3LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_F k_F}}{M^2} G_{3TT}^\perp(z, k_{F\perp}) \right], \end{aligned} \quad (\text{A24})$$

$$\begin{aligned} z\Xi_{\rho\alpha}^{T(0)}(z, k_{F\perp}; p, S) &= -p^+ \bar{n}_{[\rho} \tilde{S}_{LT\alpha]} H_{1LT}(z, k_{F\perp}) - \frac{p^+}{M} \bar{n}_{[\rho} \tilde{S}_{TT\alpha]}^{k_F} H_{1TT}^\perp(z, k_{F\perp}) \\ &\quad - \frac{p^+}{M} \bar{n}_{[\rho} \tilde{k}_{F\perp\alpha]} \left[ S_{LL} H_{1LL}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} H_{1LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} H_{1TT}^\perp(z, k_{F\perp}) \right] \\ &\quad + M\epsilon_{\perp\rho\alpha} \left[ S_{LL} H_{LL}(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} H_{LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} H_{TT}^\perp(z, k_{F\perp}) \right] \\ &\quad + \bar{n}_{[\rho} n_{\alpha]} \left[ (\tilde{k}_{F\perp} \cdot S_{LT}) H_{LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_F k_F}}{M} H_{TT}^\perp(z, k_{F\perp}) \right] \\ &\quad - \frac{M}{p^+} n_{[\rho} \tilde{k}_{F\perp\alpha]} \left[ S_{LL} H_{3LL}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_{LT}}{M} H_{3LT}^\perp(z, k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} H_{3TT}^\perp(z, k_{F\perp}) \right] \end{aligned}$$

$$- \frac{M}{p^+} n_{[\rho]} \left[ M \tilde{S}_{LT\alpha} H_{3LT}(z, k_{F\perp}) + \tilde{S}_{TT\alpha}^{k_F} H_{3TT}'^\perp(z, k_{F\perp}) \right]. \quad (\text{A25})$$

We integrate over  $d^2 k_{F\perp}$  and obtain,

$$z \Xi^{T(0)}(z; p, S) = M S_{LL} E_{LL}(z), \quad (\text{A26})$$

$$z \tilde{\Xi}^{T(0)}(z; p, S) = 0, \quad (\text{A27})$$

$$z \Xi_\alpha^{T(0)}(z; p, S) = p^+ \bar{n}_\alpha S_{LL} D_{1LL}(z) + M S_{LT\alpha} D_{LT}(z) + \frac{M^2}{p^+} n_\alpha S_{LL} D_{3LL}(z), \quad (\text{A28})$$

$$z \tilde{\Xi}_\alpha^{T(0)}(z; p, S) = - M \tilde{S}_{LT\alpha} G_{LT}(z), \quad (\text{A29})$$

$$z \Xi_{\rho\alpha}^{T(0)}(z; p, S) = - p^+ \bar{n}_{[\rho} \tilde{S}_{LT\alpha]} H_{1LT}(z) + M \varepsilon_{\perp\rho\alpha} S_{LL} H_{LL}(z) - \frac{M^2}{p^+} n_{[\rho} \tilde{S}_{LT\alpha]} H_{3LT}(z). \quad (\text{A30})$$

Again, the 4  $S_{LL}$ -dependent one-dimensional FFs are just equal to the corresponding three-dimensional FFs integrated over  $d^2 k_{F\perp}$ , while the 4  $S_{LT}$ -dependent FFs are given by Eq. (A20) for  $K_T = D_{LT}, G_{LT}, H_{1LT}$  and  $H_{3LT}$ .

We list those twist-2 FFs in table II, and those twist-3 FFs in table III. The twist-4 FFs have the same structure of those at twist-2, so we do not make a separate table. We also list them according to chiral and time-reversal properties in table IV.

TABLE II: The 18 leading twist components of the FFs for quark fragments to spin-1 hadrons

quark polarization	hadron polarization	TMD FFs	integrated over $\vec{k}_{F\perp}$	name
$U$	$U$	$D_1(z, k_{F\perp})$	$D_1(z)$	number density
	$T$	$D_{1T}^\perp(z, k_{F\perp})$	$\times$	
	$LL$	$D_{1LL}(z, k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	$LT$	$D_{1LT}^\perp(z, k_{F\perp})$	$\times$	
	$TT$	$D_{1TT}^\perp(z, k_{F\perp})$	$\times$	
$L$	$L$	$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	$T$	$G_{1T}^\perp(z, k_{F\perp})$	$\times$	
	$LT$	$G_{1LT}^\perp(z, k_{F\perp})$	$\times$	
	$TT$	$G_{1TT}^\perp(z, k_{F\perp})$	$\times$	
$T$	$U$	$H_1^\perp(z, k_{F\perp})$	$\times$	Collins function
	$T(\parallel)$	$H_{1T}(z, k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$	$H_{1T}^\perp(z, k_{F\perp})$		
	$L$	$H_{1L}^\perp(z, k_{F\perp})$	$\times$	
	$LL$	$H_{1LL}^\perp(z, k_{F\perp})$	$\times$	
	$LT$	$H_{1LT}(z, k_{F\perp}), H_{1LT}^\perp(z, k_{F\perp})$	$H_{1LT}(z)$	
	$TT$	$H_{1TT}^\perp(z, k_{F\perp}), H_{1TT}'^\perp(z, k_{F\perp})$	$\times, \times$	

#### 4. Twist-3 FFs defined via quark-gluon-quark correlator

Twist-3 components are the leading twist contributions that we obtain from  $\hat{\Xi}_\rho^{(1)}$ . There has to be one  $\bar{n}$  involved in the basic Lorentz covariants and the other(s) are from the transverse components. Since the  $\bar{n}$  component of gluon field goes into the gauge link, we only have the other three components for  $D_\rho$  thus no  $\bar{n}_\rho$ -component exists in the basic Lorentz covariants. We therefore do not have twist-3 contributions from  $\Xi_\rho^{(1)}$  or  $\tilde{\Xi}_\rho^{(1)}$ . The twist-3 contributions are obtained from  $\Xi_{\rho\alpha}^{(1)}, \tilde{\Xi}_{\rho\alpha}^{(1)}$  and  $\Xi_{\rho\alpha\beta}^{(1)}$  and are given in the following.

TABLE III: The 36 twist-3 components of the FFs for quark fragments to spin-1 hadrons

quark polarization	hadron polarization	TMD FFs	integrated over $\vec{k}_{F\perp}$
$U$	$U$	$E(z, k_{F\perp}), D^\perp(z, k_{F\perp})$	$E(z), \times$
	$L$	$D_L^\perp(z, k_{F\perp})$	$\times$
	$T$	$E_T^\perp(z, k_{F\perp}), D_T(z, k_{F\perp}), D_T^\perp(z, k_{F\perp})$	$\times, D_T(z)$
	$LL$	$E_{LL}(z, k_{F\perp}), D_{LL}^\perp(z, k_{F\perp})$	$E_{LL}(z), \times$
	$LT$	$E_{LT}^\perp(z, k_{F\perp}), D_{LT}(z, k_{F\perp}), D_{LT}^\perp(z, k_{F\perp})$	$\times, D_{LT}(z)$
	$TT$	$E_{TT}^\perp(z, k_{F\perp}), D_{TT}^\perp(z, k_{F\perp}), D_{TT}'^\perp(z, k_{F\perp})$	$\times, \times, \times$
$L$	$U$	$G^\perp(z, k_{F\perp})$	$\times$
	$L$	$E_L(z, k_{F\perp}), G_L^\perp(z, k_{F\perp})$	$E_L(z), \times$
	$T$	$E_T'^\perp(z, k_{F\perp}), G_T(z, k_{F\perp}), G_T^\perp(z, k_{F\perp})$	$\times, G_T(z)$
	$LL$	$G_{LL}^\perp(z, k_{F\perp})$	$\times$
	$LT$	$E_{LT}'^\perp(z, k_{F\perp}), G_{LT}(z, k_{F\perp}), G_{LT}^\perp(z, k_{F\perp})$	$\times, G_{LT}(z)$
	$TT$	$E_{TT}'^\perp(z, k_{F\perp}), G_{TT}^\perp(z, k_{F\perp}), G_{TT}'^\perp(z, k_{F\perp})$	$\times, \times, \times$
$T$	$U$	$H(z, k_{F\perp})$	$H(z)$
	$L$	$H_L(z, k_{F\perp})$	$H_L(z)$
	$T(\parallel)$	$H_T^\perp(z, k_{F\perp})$	$\times$
	$T(\perp)$	$H_T'^\perp(z, k_{F\perp})$	$\times$
	$LL$	$H_{LL}(z, k_{F\perp})$	$H_{LL}(z)$
	$LT$	$H_{LT}^\perp(z, k_{F\perp}), H_{LT}'^\perp(z, k_{F\perp})$	$\times, \times$
	$TT$	$H_{TT}^\perp(z, k_{F\perp}), H_{TT}'^\perp(z, k_{F\perp})$	$\times, \times$

TABLE IV: Chiral and time reversal properties of TMD FFs from quark-quark correlator

quark polarization	hadron polarization	chiral-even		chiral-odd	
		T-even	T-odd	T-even	T-odd
$U$	$U$	$D_1, D^\perp, D_3$		$E$	
	$L$		$D_L^\perp$		
	$T$		$D_{1T}^\perp, D_T, D_T^\perp, D_{3T}^\perp$		$E_T^\perp$
	$LL$	$D_{1LL}, D_{LL}^\perp, D_{3LL}$		$E_{LL}$	
	$LT$	$D_{1LT}^\perp, D_{LT}, D_{LT}^\perp, D_{3LT}^\perp$		$E_{LT}^\perp$	
	$TT$	$D_{1TT}^\perp, D_{TT}^\perp, D_{TT}'^\perp, D_{3TT}^\perp$		$E_{TT}^\perp$	
$L$	$U$		$G^\perp$		
	$L$	$G_{1L}, G_L^\perp, G_{3L}$			$E_L$
	$T$	$G_{1T}^\perp, G_T, G_T^\perp, G_{3T}^\perp$			$E_T'^\perp$
	$LL$		$G_{LL}^\perp$		
	$LT$		$G_{1LT}^\perp, G_{LT}, G_{LT}^\perp, G_{3LT}^\perp$	$E_{LT}'^\perp$	
	$TT$		$G_{1TT}^\perp, G_{TT}^\perp, G_{TT}'^\perp, G_{3TT}^\perp$	$E_{TT}'^\perp$	
$T$	$U$				$H_1^\perp, H, H_3^\perp$
	$L$			$H_{1L}^\perp, H_L, H_{3L}^\perp$	
	$T(\parallel)$			$H_{1T}, H_T^\perp, H_{3T}$	
	$T(\perp)$			$H_{1T}^\perp, H_T'^\perp, H_{3T}^\perp$	
	$LL$				$H_{1LL}^\perp, H_{LL}, H_{3LL}^\perp$
	$LT$				$H_{1LT}^\perp, H_{1LT}^\perp, H_{LT}^\perp, H_{LT}'^\perp, H_{3LT}, H_{3LT}^\perp$
	$TT$				$H_{1TT}^\perp, H_{1TT}^\perp, H_{TT}^\perp, H_{TT}'^\perp, H_{3TT}^\perp, H_{3TT}'^\perp$



For the unpolarized part, we have,

$$z\Xi_{\rho\alpha}^{U(1)}(z, k_{F\perp}; p) = -p^+ \bar{n}_\alpha k_{F\perp\rho} D_d^\perp(z, k_{F\perp}) + \dots, \quad (\text{A31})$$

$$z\tilde{\Xi}_{\rho\alpha}^{U(1)}(z, k_{F\perp}; p) = -ip^+ \bar{n}_\alpha \tilde{k}_{F\perp\rho} G_d^\perp(z, k_{F\perp}) + \dots, \quad (\text{A32})$$

$$z\Xi_{\rho\alpha\beta}^{U(1)}(z, k_{F\perp}; p) = -p^+ \left[ M \varepsilon_{\perp\rho[\alpha} \bar{n}_{\beta]} H_d(z, k_{F\perp}) - \frac{1}{M} \tilde{k}_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_d^\perp(z, k_{F\perp}) \right] + \dots. \quad (\text{A33})$$

For the vector polarization dependent part, we have,

$$z\Xi_{\rho\alpha}^{V(1)}(z, k_{F\perp}; p, S) = p^+ \bar{n}_\alpha \left\{ M \tilde{S}_{T\rho} D_{dT}(z, k_{F\perp}) + \tilde{k}_{F\perp\rho} \left[ \lambda D_{dL}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} D_{dT}^\perp(z, k_{F\perp}) \right] \right\} + \dots, \quad (\text{A34})$$

$$z\tilde{\Xi}_{\rho\alpha}^{V(1)}(z, k_{F\perp}; p, S) = -ip^+ \bar{n}_\alpha \left\{ M S_{T\rho} G_{dT}(z, k_{F\perp}) + k_{\perp\rho} \left[ \lambda G_{dL}^\perp(z, k_{F\perp}) + \frac{k_{F\perp} \cdot S_T}{M} G_{dT}^\perp(z, k_{F\perp}) \right] \right\} + \dots, \quad (\text{A35})$$

$$\begin{aligned} z\Xi_{\rho\alpha\beta}^{V(1)}(z, k_{F\perp}; p, S) = & p^+ \left\{ \lambda \left[ M g_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dL}(z, k_{F\perp}) + \frac{1}{M} k_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dL}^\perp(z, k_{F\perp}) \right] \right. \\ & - (\tilde{k}_{F\perp} \cdot S_T) \left[ \varepsilon_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dT}^\perp(z, k_{F\perp}) - \frac{1}{M^2} \tilde{k}_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp'}(z, k_{F\perp}) \right] \\ & \left. + (k_{F\perp} \cdot S_T) \left[ g_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp'}(z, k_{F\perp}) + \frac{1}{M^2} k_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp''}(z, k_{F\perp}) \right] \right\} + \dots. \end{aligned} \quad (\text{A36})$$

For tensor polarization dependent part, we have,

$$\begin{aligned} z\Xi_{\rho\alpha}^{T(1)}(z, k_{F\perp}; p, S) = & -p^+ \bar{n}_\alpha \left[ k_{F\perp\rho} S_{LL} D_{dLL}^\perp(z, k_{F\perp}) + M S_{LT\rho} D_{dLT}(z, k_{F\perp}) + k_{F\perp\rho} \frac{k_{F\perp} \cdot S_{LT}}{M} D_{dLT}^\perp(z, k_{F\perp}) \right. \\ & \left. + S_{TT\rho}^{k_F} D_{dT}^{\perp'}(z, k_{F\perp}) + k_{F\perp\rho} \frac{S_{TT}^{k_F}}{M^2} D_{dT}^\perp(z, k_{F\perp}) \right] + \dots, \end{aligned} \quad (\text{A37})$$

$$\begin{aligned} z\tilde{\Xi}_{\rho\alpha}^{T(1)}(z, k_{F\perp}; p, S) = & -ip^+ \bar{n}_\alpha \left[ \tilde{k}_{F\perp\rho} S_{LL} G_{dLL}^\perp(z, k_{F\perp}) + M \tilde{S}_{LT\rho} G_{dLT}(z, k_{F\perp}) + \frac{1}{M} \tilde{k}_{F\perp\rho} k_{F\perp} \cdot S_{LT} G_{dLT}^\perp(z, k_{F\perp}) \right. \\ & \left. + \tilde{S}_{TT\rho}^{k_F} G_{dT}^{\perp'}(z, k_{F\perp}) + \tilde{k}_{F\perp\rho} \frac{S_{TT}^{k_F}}{M^2} G_{dT}^\perp(z, k_{F\perp}) \right] + \dots, \end{aligned} \quad (\text{A38})$$

$$\begin{aligned} z\Xi_{\rho\alpha\beta}^{T(1)}(z, k_{F\perp}; p, S) = & p^+ \left\{ S_{LL} \left[ M \varepsilon_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dLL}(z, k_{F\perp}) - \frac{1}{M} \tilde{k}_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dLL}^\perp(z, k_{F\perp}) \right] \right. \\ & (k_{F\perp} \cdot S_{LT}) \left[ \varepsilon_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dT}^\perp(z, k_{F\perp}) - \frac{1}{M^2} \tilde{k}_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp'}(z, k_{F\perp}) \right] \\ & - (\tilde{k}_{F\perp} \cdot S_{LT}) \left[ g_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp'}(z, k_{F\perp}) + \frac{1}{M^2} k_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp''}(z, k_{F\perp}) \right] \\ & + \frac{S_{TT}^{k_F}}{M} \left[ \varepsilon_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dT}^\perp(z, k_{F\perp}) - \frac{1}{M^2} \tilde{k}_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp'}(z, k_{F\perp}) \right] \\ & \left. + \frac{S_{TT}^{k_F}}{M} \left[ g_{\perp\rho[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp'}(z, k_{F\perp}) + \frac{1}{M^2} k_{F\perp\rho} k_{F\perp[\alpha} \bar{n}_{\beta]} H_{dT}^{\perp''}(z, k_{F\perp}) \right] \right\} + \dots. \end{aligned} \quad (\text{A39})$$

Here, we use a subscript  $d$  to specify that they are defined via quark-gluon-quark correlator. A prime in the superscript before the  $\perp$  denotes different polarization situation, that after the  $\perp$  specifies different FF for the same polarization situation. We see that we have totally 36 FFs at twist-3 defined via quark-gluon-quark correlator. This is just the same as what we obtained from the quark-quark correlator. Among them, 18 are  $\chi$ -even and the other 18 are  $\chi$ -odd; 4 contribute to unpolarized part, 12 to vector polarized part and 20 to the tensor polarized part. We note in particular that the hermiticity in this case does not demand that the FFs defined via quark-gluon-quark correlator are real. They can have both real and imaginary parts.

## Appendix B: Twist-3 contributions to the hadronic tensor

In the two-hadron-collinear frame, the twist-3 contributions to other parts of the hadronic tensor besides  $W_{\mu\nu}^{(1)U}$  and  $W_{\mu\nu}^{(1)LL}$  given by Eqs. (4.14) and (4.19) are given by,

$$W_{\mu\nu}^{(1)L} = \frac{4\lambda}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{1}{p_1^+} \left[ -\omega_{\mu\nu}(\tilde{k}) D_L^\perp + \tilde{\omega}_{\mu\nu}(k) G_L^\perp \right] \bar{D}_1 + \frac{2M_1 c_2^q}{M_2 p_1^+} \left[ 2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} H_L + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} E_L \right] \bar{H}_1^\perp \right\}$$

$$\begin{aligned}
& + \frac{1}{p_2^-} G_{1L} [\omega_{\mu\nu}(k') \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') \bar{G}^\perp] - \frac{2M_2 c_2^q}{M_1 p_2^-} H_{1L}^\perp [2(\tilde{k}_n - \tilde{k}_{\bar{n}})_{[\mu\nu]} \bar{H} + i(\tilde{k}_n - \tilde{k}_{\bar{n}})_{[\mu\nu]} \bar{E}] \\
& - \frac{\sqrt{2}}{Q} \left[ \tilde{\omega}_{\mu\nu}(k', k) G_{1L} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (k_\perp^2 \tilde{k}'_{\bar{n}} + \tilde{k}_\perp \cdot k'_\perp k'_n + k'_\perp \cdot k_\perp k'_n)_{[\mu\nu]} H_{1L}^\perp \bar{H}_1^\perp \right], \tag{B1}
\end{aligned}$$

$$\begin{aligned}
W_{\mu\nu}^{(1)T} = & \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{k_\perp \cdot S_T}{M_1} \left( \frac{1}{p_1^+} [\omega_{\mu\nu}(-\tilde{k}, \tilde{k}) D_T^\perp + \tilde{\omega}_{\mu\nu}(k) G_T^\perp] \bar{D}_1 + \frac{1}{p_2^-} G_{1T} [\tilde{\omega}_{\mu\nu}(k') \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') \bar{G}^\perp] \right. \right. \\
& + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} H_T'^\perp + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} E_T'^\perp] \bar{H}_1^\perp - \frac{2M_2 c_2^q}{M_1 p_2^-} H_{1T}^\perp [2(\tilde{k}_n - \tilde{k}_{\bar{n}})_{[\mu\nu]} \bar{H} + i(\tilde{k}_n - \tilde{k}_{\bar{n}})_{[\mu\nu]} \bar{E}] \\
& \left. - \frac{\sqrt{2}}{Q} [\tilde{\omega}_{\mu\nu}(k', k) G_{1T} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (k_\perp^2 \tilde{k}'_{\bar{n}} + \tilde{k}_\perp \cdot k'_\perp k'_n + k'_\perp \cdot k_\perp k'_n)_{[\mu\nu]} H_{1T}^\perp \bar{H}_1^\perp] \right\} \\
& + \frac{\tilde{k}_\perp \cdot S_T}{M_1} \left( \frac{1}{p_2^-} D_{1T}^\perp [-\omega_{\mu\nu}(k') \bar{D}^\perp - \tilde{\omega}_{\mu\nu}(\tilde{k}') \bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(k'_n - k'_{\bar{n}})_{[\mu\nu]} H_T'^\perp + i(k'_n - k'_{\bar{n}})_{[\mu\nu]} E_T'^\perp] \bar{H}_1^\perp + \frac{\sqrt{2}}{Q} \omega_{\mu\nu}(k', k) D_{1T}^\perp \bar{D}_1 \right) \\
& - \frac{2M_2 c_2^q}{p_2^-} H_{1T} [2(\tilde{S}_n - \tilde{S}_{\bar{n}})_{[\mu\nu]} \bar{H} + i(\tilde{S}_n - \tilde{S}_{\bar{n}})_{[\mu\nu]} \bar{E}] + \frac{M_1}{p_1^+} [-\omega_{\mu\nu}(\tilde{S}) D_T + \tilde{\omega}_{\mu\nu}(S) G_T] \bar{D}_1 \\
& - \frac{4\sqrt{2}c_2^q}{M_2 Q} [-\tilde{k}'_\perp \cdot S_\perp (k_{\bar{n}} + k'_n)_{[\mu\nu]} + \tilde{k}'_\perp \cdot k_\perp S_{\bar{n}[\mu\nu]} + k_\perp \cdot S_\perp \tilde{k}'_{\bar{n}[\mu\nu]} + k'_\perp \cdot S_\perp \tilde{k}'_{n[\mu\nu]}] H_{1T} \bar{H}_1^\perp], \tag{B2}
\end{aligned}$$

$$\begin{aligned}
W_{\mu\nu}^{(1)LT} = & \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{k_\perp \cdot S_{LT}}{M_1} \left( \frac{1}{p_1^+} [\omega_{\mu\nu}(k) D_{LT}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}) G_{LT}^\perp] \bar{D}_1 - \frac{1}{p_2^-} [\omega_{\mu\nu}(k') D_{1LT}^\perp \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') D_{1LT}^\perp \bar{G}^\perp] \right. \right. \\
& + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(k'_n - k'_{\bar{n}})_{[\mu\nu]} H_{LT}^\perp + i(k'_n - k'_{\bar{n}})_{[\mu\nu]} E_{LT}^\perp] \bar{H}_1^\perp - \frac{2M_2 c_2^q}{M_1 p_2^-} [2(k_n - k_{\bar{n}})_{[\mu\nu]} H_{1LT}^\perp \bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]} H_{1LT}^\perp \bar{E}] \\
& \left. + \frac{\sqrt{2}}{Q} [\omega_{\mu\nu}(k', k) D_{1LT}^\perp \bar{D}_1 - \frac{4}{M_1 M_2} \omega_{\mu\nu}^{(n)}(k, k') H_{1LT}^\perp \bar{H}_1^\perp] \right\} \\
& + \frac{\tilde{k}_\perp \cdot S_{LT}}{M_1} \left( \frac{1}{p_2^-} G_{1LT}^\perp [\tilde{\omega}_{\mu\nu}(k') \bar{D}^\perp + \omega_{\mu\nu}(\tilde{k}') \bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} H_{LT}'^\perp + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} E_{LT}'^\perp] \bar{H}_1^\perp - \frac{\sqrt{2}}{Q} \tilde{\omega}_{\mu\nu}(k', k) G_{1LT}^\perp \bar{D}_1 \right) \\
& + \frac{M_1}{p_1^+} [\omega_{\mu\nu}(S_{LT}) D_{LT} + \tilde{\omega}_{\mu\nu}(\tilde{S}_{LT}) G_{LT}] \bar{D}_1 - \frac{2M_2 c_2^q}{p_2^-} [2(S_{LTn} - S_{LT\bar{n}})_{[\mu\nu]} H_{1LT} \bar{H} + i(S_{LTn} - S_{LT\bar{n}})_{[\mu\nu]} H_{1LT} \bar{E}] \\
& - \frac{4\sqrt{2}c_2^q}{M_2 Q} (k_\perp \cdot S_{LT} k'_{\bar{n}} - k'_\perp \cdot S_{LT} k_{\bar{n}} + k_\perp \cdot k'_\perp S_{LT\bar{n}} + k_\perp'^2 S_{LTn})_{[\mu\nu]} H_{1LT} \bar{H}_1^\perp], \tag{B3}
\end{aligned}$$

$$\begin{aligned}
W_{\mu\nu}^{(1)TT} = & \frac{4}{z_1 z_2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{S_{TT}^{kk}}{M_1^2} \left( \frac{1}{p_1^+} [\omega_{\mu\nu}(k) D_{TT}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}) G_{TT}^\perp] \bar{D}_1 - \frac{1}{p_2^-} [\omega_{\mu\nu}(k') D_{1TT}^\perp \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') D_{1TT}^\perp \bar{G}^\perp] \right. \right. \\
& + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(k'_n - k'_{\bar{n}})_{[\mu\nu]} H_{TT}^\perp + i(k'_n - k'_{\bar{n}})_{[\mu\nu]} E_{TT}^\perp] \bar{H}_1^\perp - \frac{2M_2 c_2^q}{M_1 p_2^-} [2(k_n - k_{\bar{n}})_{[\mu\nu]} H_{1TT}^\perp \bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]} S_{TT}^{kk} H_{1TT}^\perp \bar{E}] \\
& \left. + \frac{\sqrt{2}}{Q} [\omega_{\mu\nu}(k', k) D_{1TT}^\perp \bar{D}_1 - \frac{4}{M_1 M_2} \omega_{\mu\nu}^{(n)}(k, k') H_{1TT}^\perp \bar{H}_1^\perp] \right\} \\
& + \frac{S_{TT}^{kk}}{M_1^2} \left( \frac{1}{p_2^-} G_{1TT}^\perp [\tilde{\omega}_{\mu\nu}(k') \bar{D}^\perp + \omega_{\mu\nu}(\tilde{k}') \bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} H_{TT}'^\perp + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} E_{TT}'^\perp] \bar{H}_1^\perp - \frac{\sqrt{2}}{Q} \tilde{\omega}_{\mu\nu}(k', k) G_{1TT}^\perp \bar{D}_1 \right) \\
& + \frac{1}{p_1^+} [\omega_{\mu\nu}(S_{TT}^k) D_{TT}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{S}_{TT}^k) G_{TT}^\perp] \bar{D}_1 - \frac{2M_2 c_2^q}{p_2^- M_1} [2(S_{TTn}^k - S_{TT\bar{n}}^k)_{[\mu\nu]} H_{1TT}^\perp \bar{H} + i(S_{TTn}^k - S_{TT\bar{n}}^k)_{[\mu\nu]} H_{1TT}^\perp \bar{E}] \\
& - \frac{4\sqrt{2}c_2^q M_1^2}{M_2 Q} (S_{TT}^{kk} k'_{\bar{n}} - S_{TT}^{kk'} k_{\bar{n}} + k_\perp \cdot k'_\perp S_{TT\bar{n}}^k + k_\perp'^2 S_{TTn}^k)_{[\mu\nu]} H_{1TT}^\perp \bar{H}_1^\perp]. \tag{B4}
\end{aligned}$$

Transforming them into the Helicity-GJ-frame, we obtain from Eqs. (B1-B4) the contributions at twist-3 and they take exactly the same form as given in these equations. However, we obtain also additional twist-3 contributions from the twist-2 parts

given by Eqs. (4.4-4.13). The corresponding terms for the unpolarized part is given by Eq. (4.23). Other parts are given in the following.

$$\delta W_{\mu\nu}^{(1)L} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \lambda \left\{ (c_3^q q_{\bar{n}[\mu\nu]} - ic_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1L} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} k_\perp \cdot (q_\perp - \tilde{k}'_\perp) k'_{\bar{n}[\mu\nu]} H_{1L}^\perp \bar{H}_1^\perp \right\}, \quad (\text{B5})$$

$$\begin{aligned} \delta W_{\mu\nu}^{(1)T} = & \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{k_\perp \cdot S_\perp}{M_1} [(c_3^q q_{\bar{n}[\mu\nu]} - ic_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1T}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} k_\perp \cdot (q_\perp - \tilde{k}'_\perp) k'_{\bar{n}[\mu\nu]} H_{1T}^\perp \bar{H}_1^\perp] \right. \\ & \left. - \frac{\tilde{k}_\perp \cdot S_\perp}{M_1} (c_1^q q_{\bar{n}[\mu\nu]} - ic_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1T}^\perp \bar{D}_1 + \frac{4c_2^q}{M_2} (k_\perp \cdot S_\perp \tilde{k}'_{\bar{n}} + k_\perp \cdot \tilde{k}'_\perp S_{\bar{n}} - \tilde{k}'_\perp \cdot S_\perp k_{\bar{n}})_{[\mu\nu]} H_{1T} \bar{H}_1^\perp \right\}, \end{aligned} \quad (\text{B6})$$

$$\delta W_{\mu\nu}^{(1)LL} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) S_{LL} \left\{ -(c_1^q q_{\bar{n}[\mu\nu]} - ic_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1LL} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (k_\perp^2 k'_{\bar{n}} + k_\perp'^2 k_{\bar{n}})_{[\mu\nu]} H_{1LL}^\perp \bar{H}_1^\perp \right\}, \quad (\text{B7})$$

$$\begin{aligned} \delta W_{\mu\nu}^{(1)LT} = & \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{k_\perp \cdot S_{LT}}{M_1} [(c_1^q q_{\bar{n}[\mu\nu]} - ic_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (k_\perp^2 k'_{\bar{n}} + k_\perp'^2 k_{\bar{n}})_{[\mu\nu]} H_{1LT}^\perp \bar{H}_1^\perp] \right. \\ & \left. + \frac{\tilde{k}_\perp \cdot S_{LT}}{M_1} (c_3^q q_{\bar{n}[\mu\nu]} - ic_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_2} (q_\perp \cdot k'_\perp S_{LT\bar{n}} + k_\perp \cdot S_{LT} k'_{\bar{n}} - k'_\perp \cdot S_{LT} k_{\bar{n}})_{[\mu\nu]} H_{1LT} \bar{H}_1^\perp \right\}, \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} \delta W_{\mu\nu}^{(1)TT} = & \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{S_{TT}^{kk}}{M_1^2} [(c_1^q q_{\bar{n}[\mu\nu]} - ic_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1TT}^\perp \bar{D}_1 + (k_\perp^2 k'_{\bar{n}} + k_\perp'^2 k_{\bar{n}})_{[\mu\nu]} H_{1TT}^\perp \bar{H}_1^\perp] \right. \\ & \left. + \frac{S_{TT}^{kk}}{M_1^2} (c_3^q q_{\bar{n}[\mu\nu]} - ic_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (q_\perp \cdot k'_\perp S_{TT\bar{n}}^k + S_{TT}^{kk} k'_{\bar{n}} - S_{TT}^{kk'} k_{\bar{n}})_{[\mu\nu]} H_{1TT} \bar{H}_1^\perp \right\}. \end{aligned} \quad (\text{B9})$$

### Appendix C: Twist-3 contributions to the structure functions

In the partonic picture at the LO pQCD, 36 of the 81 structure functions for  $e^+e^- \rightarrow V\pi X$  have twist-3 contributions. We list the results in this appendix in the following.

$$F_{1U2}^{\cos\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 D^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_1 \bar{D}^{\perp'}], \quad (\text{C1})$$

$$F_{2U2}^{\cos\varphi} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_1 D^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_1 \bar{D}^{\perp'}] + 4c_2^q C[M_1 \bar{w}_1 H z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_1^\perp \bar{H}^{\perp'}] \right\}, \quad (\text{C2})$$

$$\tilde{F}_{1U2}^{\sin\varphi} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[(M_1 w_1 G^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_1 \bar{G}^\perp)] + 2c_2^q C[(M_1 \bar{w}_1 E z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_1^\perp \bar{E}] \right\}, \quad (\text{C3})$$

$$\tilde{F}_{2U2}^{\sin\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 G^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_1 \bar{G}^\perp], \quad (\text{C4})$$

$$\tilde{F}_{1L2}^{\cos\varphi} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_1 G_L^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 G_{1L} \bar{D}^{\perp'}] + 2c_2^q C[-M_1 \bar{w}_1 E_L z_2 \bar{H}_1^\perp + M_2 w_1 z_1 H_{1L}^\perp \bar{E}] \right\}, \quad (\text{C5})$$

$$\tilde{F}_{2L2}^{\cos\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 G_L^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 G_{1L} \bar{D}^{\perp'}], \quad (\text{C6})$$

$$F_{1L2}^{\sin\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[-M_1 w_1 D_L^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 G_{1L} \bar{G}^\perp], \quad (\text{C7})$$

$$F_{2L2}^{\sin\varphi} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[-M_1 w_1 D_L^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 G_{1L} \bar{G}^\perp] + 4c_2^q C[M_1 \bar{w}_1 H_L z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_{1L}^\perp \bar{H}^{\perp'}] \right\}, \quad (\text{C8})$$

$$\tilde{F}_{1T2}^{\cos\varphi_S} = \frac{4c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[2M_1 \mathcal{G}_T^\perp z_2 \bar{D}_1 + M_2 w_2 z_1 (G_{1T}^\perp \bar{D}^{\perp'} + D_{1T}^\perp \bar{G}^\perp)] + c_2^q C[-2M_1 w_2 E_T^\perp z_2 \bar{H}_1^\perp + M_2 z_1 \mathcal{H}_{1T}^\perp \bar{E}] \right\}, \quad (\text{C9})$$

$$\tilde{F}_{2T2}^{\cos\varphi_S} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 \mathcal{G}_T^\perp z_2 \bar{D}_1 + M_2 \frac{w_2}{2} z_1 (G_{1T}^\perp \bar{D}^{\perp'} + D_{1T}^\perp \bar{G}^\perp)], \quad (\text{C10})$$

$$F_{1T2}^{\sin\varphi_S} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[-M_1 \mathcal{D}_T^\perp z_2 \bar{D}_1 + M_2 \frac{w_2}{2} z_1 (D_{1T}^\perp \bar{D}^{\perp'} - G_{1T}^\perp \bar{G}^\perp)], \quad (\text{C11})$$

$$F_{2T2}^{\sin\varphi_S} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[-M_1 \mathcal{D}_T^\perp z_2 \bar{D}_1 + M_2 \frac{w_2}{2} z_1 (D_{1T}^\perp \bar{D}^{\perp'} - G_{1T}^\perp \bar{G}^\perp)] + 4c_2^q C[M_1 \frac{w_2}{2} H_T^{\perp-} z_2 \bar{H}_1^\perp - M_2 z_1 \mathcal{H}_{1T}^\perp \bar{H}_1^{\perp'}] \right\}, \quad (C12)$$

$$\tilde{F}_{1T2}^{\cos(\varphi_S-2\varphi)} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_3 G_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (G_{1T}^\perp \bar{D}^{\perp'} - D_{1T}^\perp \bar{G}^\perp)] + 2c_2^q C[-M_1 w_4 E_T^{\perp+} z_2 \bar{H}_1^\perp + M_2 w_3 z_1 H_{1T}^\perp \bar{E}] \right\}, \quad (C13)$$

$$\tilde{F}_{2T2}^{\cos(\varphi_S-2\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 w_3 G_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (G_{1T}^\perp \bar{D}^{\perp'} - D_{1T}^\perp \bar{G}^\perp)], \quad (C14)$$

$$F_{1T2}^{\sin(\varphi_S-2\varphi)} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 w_3 D_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (D_{1T}^\perp \bar{D}^{\perp'} + G_{1T}^\perp \bar{G}^\perp)], \quad (C15)$$

$$F_{2T2}^{\sin(\varphi_S-2\varphi)} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_3 D_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (D_{1T}^\perp \bar{D}^{\perp'} + G_{1T}^\perp \bar{G}^\perp)] + 4c_2^q C[-M_1 w_4 H_T^{\perp+} z_2 \bar{H}_1^\perp + M_2 w_3 z_1 H_{1T}^\perp \bar{H}^{\perp'}] \right\}, \quad (C16)$$

$$F_{1LL2}^{\cos\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 D_{LL}^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_{1LL} \bar{D}^{\perp'}], \quad (C17)$$

$$F_{2LL2}^{\cos\varphi} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_1 D_{LL}^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_{1LL} \bar{D}^{\perp'}] + 4c_2^q C[M_1 \bar{w}_1 H_{LL} z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_{1LL}^\perp \bar{H}^{\perp'}] \right\}, \quad (C18)$$

$$\tilde{F}_{2LL2}^{\sin\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 G_{LL}^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_{1LL} \bar{G}^\perp], \quad (C19)$$

$$\tilde{F}_{1LL2}^{\sin\varphi} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_1 G_{LL}^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_{1LL} \bar{G}^\perp] + 2c_2^q C[M_1 \bar{w}_1 E_{LL} z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_{1LL}^\perp \bar{E}] \right\}, \quad (C20)$$

$$F_{1LT2}^{\cos\varphi_{LT}} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 \mathcal{D}_{LT}^\perp z_2 \bar{D}_1 - M_2 \frac{w_2}{2} z_1 (D_{1LT}^\perp \bar{D}^{\perp'} + G_{1LT}^\perp \bar{G}^\perp)], \quad (C21)$$

$$F_{2LT2}^{\cos\varphi_{LT}} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 \mathcal{D}_{LT}^\perp z_2 \bar{D}_1 - M_2 \frac{w_2}{2} z_1 (D_{1LT}^\perp \bar{D}^{\perp'} + G_{1LT}^\perp \bar{G}^\perp)] - 4c_2^q C[M_1 \frac{w_2}{2} H_{LT}^{\perp+} z_2 \bar{H}_1^\perp + M_2 z_1 \mathcal{H}_{1LT}^\perp \bar{H}_1^{\perp'}] \right\}, \quad (C22)$$

$$\tilde{F}_{1LT2}^{\sin\varphi_{LT}} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 \mathcal{G}_{LT}^\perp z_2 \bar{D}_1 - M_2 \frac{w_2}{2} z_1 (G_{1LT}^\perp \bar{D}^{\perp'} - D_{1LT}^\perp \bar{G}^\perp)] + c_2^q C[-M_1 w_2 E_{LT}^{\perp+} z_2 \bar{H}_1^\perp - 2M_2 z_1 \mathcal{H}_{1LT}^\perp \bar{E}] \right\}, \quad (C23)$$

$$\tilde{F}_{2LT2}^{\sin\varphi_{LT}} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 \mathcal{G}_{LT}^\perp z_2 \bar{D}_1 - M_2 \frac{w_2}{2} z_1 (G_{1LT}^\perp \bar{D}^{\perp'} - D_{1LT}^\perp \bar{G}^\perp)], \quad (C24)$$

$$F_{1LT2}^{\cos(\varphi_{LT}-2\varphi)} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 w_3 D_{LT}^\perp z_2 \bar{D}_1 + M_2 w_4 z_1 (D_{1LT}^\perp \bar{D}^{\perp'} - G_{1LT}^\perp \bar{G}^\perp)], \quad (C25)$$

$$F_{2LT2}^{\cos(\varphi_{LT}-2\varphi)} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_3 D_{LT}^\perp z_2 \bar{D}_1 + M_2 w_4 z_1 (D_{1LT}^\perp \bar{D}^{\perp'} - G_{1LT}^\perp \bar{G}^\perp)] + 4c_2^q C[M_1 w_4 H_{LT}^{\perp-} z_2 \bar{H}_1^\perp - M_2 w_3 z_1 H_{1LT}^\perp \bar{H}^{\perp'}] \right\}, \quad (C26)$$

$$\tilde{F}_{1LT2}^{\sin(\varphi_{LT}-2\varphi)} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[-M_1 w_3 G_{LT}^\perp z_2 \bar{D}_1 + M_2 w_4 z_1 (G_{1LT}^\perp \bar{D}^{\perp'} + D_{1LT}^\perp \bar{G}^\perp)] - 2c_2^q C[M_1 w_4 E_{LT}^{\perp-} z_2 \bar{H}_1^\perp - M_2 w_3 z_1 H_{1LT}^\perp \bar{E}] \right\}, \quad (C27)$$

$$\tilde{F}_{2LT2}^{\sin(\varphi_{LT}-2\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[-M_1 w_3 G_{LT}^\perp z_2 \bar{D}_1 + M_2 w_4 z_1 (G_{1LT}^\perp \bar{D}^{\perp'} + D_{1LT}^\perp \bar{G}^\perp)], \quad (C28)$$

$$F_{1TT2}^{\cos(2\varphi_{TT}-\varphi)} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[-M_1 w_1 \mathcal{D}_{TT}^\perp z_2 \bar{D}_1 - M_2 z_1 (w_3 \bar{w}_1 D_{1TT}^\perp \bar{D}^{\perp'} - w_5 G_{1TT}^\perp \bar{G}^\perp)], \quad (C29)$$

$$F_{2TT2}^{\cos(2\varphi_{TT}-\varphi)} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[-M_1 w_1 \mathcal{D}_{TT}^\perp z_2 \bar{D}_1 - M_2 z_1 (w_3 \bar{w}_1 D_{1TT}^\perp \bar{D}^{\perp'} - w_5 G_{1TT}^\perp \bar{G}^\perp)] + 4c_2^q C[M_1 (w_6 H_{TT}^\perp z_2 \bar{H}_1^\perp - w_8 H_{TT}'^\perp \bar{H}_1^\perp) + M_2 w_1 z_1 \mathcal{H}_{1TT}^\perp \bar{H}^{\perp'}] \right\}, \quad (C30)$$

$$\tilde{F}_{1TT2}^{\sin(2\varphi_{TT}-\varphi)} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[-M_1 w_1 \mathcal{G}_{TT}^\perp z_2 \bar{D}_1 + M_2 z_1 (w_6 G_{1TT}^\perp \bar{D}^{\perp'} + w_3 \bar{w}_1 D_{1TT}^\perp \bar{G}^\perp)] + 2c_2^q C[M_1 z_2 (w_6 E_{TT}^\perp \bar{H}_1^\perp - w_8 E_{TT}'^\perp \bar{H}_1^\perp) + M_2 w_1 z_1 \mathcal{H}_{1TT}^\perp \bar{E}] \right\}, \quad (C31)$$

$$\tilde{F}_{2TT2}^{\sin(2\varphi_{TT}-\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[-M_1 w_1 \mathcal{G}_{TT}^\perp z_2 \bar{D}_1 + M_2 z_1 (w_6 G_{1TT}^\perp \bar{D}^{\perp'} + w_3 \bar{w}_1 D_{1TT}^\perp \bar{G}^\perp)], \quad (C32)$$

$$F_{1TT2}^{\cos(2\varphi_{TT}-3\varphi)} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 w_9 D_{TT}^\perp z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 (D_{1TT}^\perp \bar{D}^{\perp'} - G_{1TT}^\perp \bar{G}^\perp)], \quad (C33)$$



$$F_{2TT2}^{\cos(2\varphi_{TT}-3\varphi)} = \frac{4c_1^e}{z_1 z_2 Q} \left\{ c_1^q C[M_1 w_9 D_{TT}^\perp z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 (D_{1TT}^\perp \bar{D}^{\perp'} - G_{1TT}^\perp \bar{G}^\perp)] + 4c_2^q C[M_1 \frac{w_7}{2} H_{TT}^{\perp-} z_2 \bar{H}_1^\perp - M_2 w_9 z_1 H_{1TT}^\perp \bar{H}^{\perp'}] \right\}, \quad (C34)$$

$$\tilde{F}_{1TT2}^{\sin(2\varphi_{TT}-3\varphi)} = \frac{8c_3^e}{z_1 z_2 Q} \left\{ c_1^q C[-M_1 w_9 G_{TT}^\perp z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 (G_{1TT}^\perp \bar{D}^{\perp'} + D_{1TT}^\perp \bar{G}^\perp)] - c_2^q C[M_1 w_7 E_{TT}^{\perp-} z_2 \bar{H}_1^\perp - 2M_2 w_9 z_1 H_{1TT}^\perp \bar{E}] \right\}, \quad (C35)$$

$$\tilde{F}_{2TT2}^{\sin(2\varphi_{TT}-3\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[-M_1 w_9 G_{TT}^\perp z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 D_{1TT}^\perp \bar{G}^\perp]. \quad (C36)$$

Here, just as for the  $S_T$ - and  $S_{LT}$ -dependent FFs given by Eq. (A20), for  $S_{TT}$ -dependent  $K$ , we define,

$$\mathcal{K}_{TT}^\perp(z, k_\perp) = K_{TT}^{\perp'}(z, k_\perp) + \frac{k_\perp^2}{2M_1^2} K_{TT}^\perp(z, k_\perp), \quad (C37)$$

for  $K = D, G$  or  $H$ . Also,  $K_\sigma^{\perp\pm} = K_\sigma^\perp \pm K_\sigma^{\perp'}$ , for all different  $K$ 's and polarization  $\sigma$ 's, and for the leading twist involved combinations,

$$\bar{D}^{\perp'} = z_2 \bar{D}_1 - \bar{D}^\perp, \quad \bar{H}^{\perp'} = \bar{H} - \bar{w}_0 z_2 \bar{H}_1^\perp. \quad (C38)$$

Besides the  $w$ 's given by Eqs. (5.2-5.6) and in the text in Sec. V, we have also introduced the scalar weights defined as,

$$w_3 = \frac{1}{2}w_0 - w_1^2, \quad w_4 = \frac{1}{2}w_2 - w_1 \bar{w}_1, \quad w_5 = w_1 w_2 - w_0 w_2 + \frac{1}{2}w_0 w_1, \quad (C39)$$

$$w_6 = w_1 w_2 - \frac{1}{2}w_0 \bar{w}_1, \quad w_7 = 4w_1^2 \bar{w}_1 - 2w_1 w_2 - w_0 \bar{w}_1, \quad w_8 = 4w_1^2 \bar{w}_1 - 2w_1 w_2 - \frac{1}{2}w_0 \bar{w}_1, \quad w_9 = (2w_1^2 - \frac{3}{2}w_0)w_1. \quad (C40)$$

They are all scalar functions of  $k_\perp$ ,  $k_\perp'$  and  $p_{2T}$ .

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  - [24] One can similarly define whether a basic Lorentz tensor  $h^{\mu\nu}$  is time reversal  $T$ -even,  $h_{\mu\nu}(q^\mathcal{T}, p_1^\mathcal{T}, S^\mathcal{T}, p_2^\mathcal{T}) = h^{\mu\nu}(q, p_1, S, p_2)$ ,

- or  $T$ -odd,  $h_{\mu\nu}(q^\mathcal{T}, p_1^\mathcal{T}, S^\mathcal{T}, p_2^\mathcal{T}) = -h^{\mu\nu}(q, p_1, S, p_2)$ , where  $a^\mathcal{T}$  denotes the  $a$  under time reversal. We note that for  $a = p, S_{LL}, S_{LT}$  or  $S_{TT}$ ,  $a^\mathcal{T} = a^\mathcal{P}$ , while for the polarization vector  $S, S^\mathcal{T} = -S^\mathcal{P}$ . Hence for the symmetric tensors, the unpolarized,  $S_{LL}, S_{LT}$  and  $S_{TT}$ -dependent parts have the same  $P$  and  $T$  behavior, while the  $S$ -dependent part have different  $P$  and  $T$  behaviors. For the anti-symmetric parts, the unpolarized,  $S_{LL}, S_{LT}$  and  $S_{TT}$ -dependent parts have different  $P$  and  $T$  behaviors, while the  $S$ -dependent part have the same  $P$  and  $T$  behaviors.
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